A Theoretical Framework for Measuring Attractiveness by a Categorical Based Evaluation Technique (MACBETH)

Carlos A. Bana e Costa[1]   and   Jean-Claude Vansnick[2]

[1] Technical University of Lisbon, IST, Dpt. Civil/CESUR, Av. Rovisco Pais, 1000, Lisbon, PORTUGAL
[2] University of Mons-Hainaut, F.S.E.S., Place du Parc, 20, 7000, Mons, BELGIUM

Abstract. This paper presents a theoretical framework for the macbeth approach. Taking as a starting point the measurement rules used for numerically representing the qualitative (categorical) preference information assessed by the questioning procedure of macbeth, we study the existence of a solution for our particular representation problem and we briefly discuss its unicity.

Introduction

Based on judgments of an evaluator D about the attractiveness of the elements of a finite set A of alternatives, MACBETH (**M**easuring **A**ttractiveness by a **C**ategorical **B**ased **E**valuation **T**echnique) is an approach to guide the construction of an interval scale for which the idea of difference of attractiveness is meaningful, that is for the construction of a numerical scale $v : A \rightarrow \Re : a \rightarrow v(a)$ which not only satisfies the

Ordinal condition:

- $\forall a, b \in A$, $v(a) > v(b)$ if and only if D judges a more attractive than b,

but also satisfies the

Cardinal condition:

- with a more attractive than b and c more attractive than d, the ratio

$$\frac{v(a) - v(b)}{v(c) - v(d)}$$

expresses the relative strength for D of the difference of attractiveness between a and b taking as unit of reference the difference of attractiveness between c and d.

To derive a scale satisfying these conditions the basic idea of MACBETH is:

1º) in a first stage, to use a very simple questioning procedure which involves only two actions in each question and to assign, to each element a of A, a real number $\mu(a)$ on the basis of straightforward rules for quantifying the preference information given by D;

2º) in a second stage, to discuss with D about the cardinality of the scale μ constructed in the first stage: does this scale satisfy the cardinal condition?

MACBETH was first proposed in [Bana e Costa and Vansnick, 1994] and two applications in decision-aiding are described in [Bana e Costa and Vansnick, 1996]. This new paper presents a theoretical framework for our approach. It includes three paragraphs: in the first, we review the questioning procedure and indicate the measurement rules used in MACBETH, while the second and third are respectively devoted to the discussion of the existence and of the unicity of a solution for the representation problem we are addressing.

1. Questioning procedure and measurement rules

1.1. Questioning procedure

In the questioning procedure of MACBETH, an evaluator D is asked to make semantic judgments about the (subjective) difference of attractiveness between actions of A:

$\forall (a,b) \in A \times A$ with $a \neq b$

	yes	no
Is a more attractive than b?	☐	☐

and, if the answer is yes:

Is the difference of attractiveness between a and b

very weak ?	☐
weak ?	☐
moderate?	☐
strong ?	☐
very strong ?	☐
extreme ?	☐

Concerning the responses of D, the following terminology and notations are adopted: