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A NEW COMBINATIVE DISTANCE-BASED ASSESSMENT (CODAS) METHOD FOR MULTI-CRITERIA DECISION-MAKING

Abstract. *A key factor to attain success in any discipline, especially in a field which requires handling large amounts of information and knowledge, is decision making. Most real-world decision-making problems involve a great variety of factors and aspects that should be considered. Making decisions in such environments can often be a difficult operation to perform. For this reason, we need multi-criteria decision-making (MCDM) methods and techniques, which can assist us for dealing with such complex problems. The aim of this paper is to present a new* ***CO****mbinative* ***D****istance-based* ***AS****sessment (CODAS) method to handle MCDM problems. To determine the desirability of an alternative, this method uses the Euclidean distance as the primary and the Taxicab distance as the secondary measure, and these distances are calculated according to the negative-ideal point. The alternative which has greater distances is more desirable in the CODAS method. Some numerical examples are used to illustrate the process of the proposed method. We also perform a comparative sensitivity analysis to examine the results of CODAS and compare it by some existing MCDM methods. These analyses show that the proposed method is efficient, and the results are stable.*

Keywords: *Multi-criteria decision-making, MCDM, MADM, Euclidean distance, Taxicab distance, CODAS.*

JEL Classification: C02, C44, C61, C63, L6

1. Introduction

Multi-criteria decision-making (MCDM) is one of the most active fields of interdisciplinary research in management science and operations research (Ho *et al.,* 2010). Multi-attribute decision-making (MADM) and multi-objective decision-making (MODM) are two branches in MCDM. MADM usually involves the discrete decision variables and a limited number of alternatives for evaluation (Jato-Espino *et al.,* 2014). MODM is concerned with identifying the best choice from an infinite set of alternatives under a set of constraints. Each criterion in MODM is associated with an objective, whereas in MADM each criterion is associated with a discrete attribute (Kabir *et al.,* 2014). However, MADM and MCDM have been used to refer the same class of problems in the recent years. In the following, we also use the term MCDM to refer multi-attribute decision-making problems. Fundamentally, intrinsic properties of MCDM make it appealing and practically useful. Some of these properties described by Belton and Stewart (Belton and Stewart, 2002) are as follows: (1) ‘‘MCDM seeks to take explicit account of multiple, conflicting criteria’’, (2) it helps to structure the management problem, (3) it provides a model that can serve as a focus for discussion, and (4) it offers a process that leads to rational, justifiable, and explainable decisions.

Many MCDM methods and techniques have been proposed by researchers in the past decades. Some of the most important ones are weighted sum model (WSM) (Fishburn, 1967), weighted product model (WPM) (Miller and Starr, 1969), weighted aggregated sum product assessment (WASPAS) (Zavadskas *et al.,* 2012), analytical hierarchy process (AHP) (Satty, 1990), ELECTRE (ELimination Et Choix Traduisant la REalité) (Roy, 1968), technique for order of preference by similarity to ideal solution (TOPSIS) (Hwang and Yoon, 1981), preference ranking organization method for enrichment of evaluations (PROMETHEE) (Brans and Vincke, 1985), complex proportional assessment (COPRAS) (Zavadskas and Kaklauskas, 1996), VIKOR (VIseKriterijumska Optimizacija I Kompromisno Resenje) (Opricovic, 1998), MULTIMOORA (multi-objective optimization by ratio analysis plus the full multiplicative form) (Brauers and Zavadskas, 2010), additive ratio assessment (ARAS) (Zavadskas and Turskis, 2010) and evaluation based on distance from average solution (EDAS) (Keshavarz Ghorabaee *et al.,* 2015). WSM is probably the most commonly used approach. This method defines the optimal alternative based on the ‘additive utility’ assumption. WPM is very similar to the WSM. This method uses the multiplication of powered weighted ratios (performances) instead of summation of weighted ratios which considered in WSM. WASPAS method was proposed based on the combination of WSM and WPM methods, and has the advantages of both of them. This method has been applied in many real-world MCDM problems (Vafaeipour *et al.,* 2014; Džiugaitė-Tumėnienė and Lapinskaitė, 2014; Petkovic *et al.,* 2015). The AHP, which was proposed by Saaty (Satty, 1981), is based on preferences or weights of importance of criteria and alternatives with respect to the hierarchical structure of them. We have three levels in the structure of the AHP method. First level is related to the goal of the problem, second level corresponds to the criteria, and third level shows the alternatives. This method involves pair-wise comparisons and therefore is time-consuming when we have numerous criteria and/or alternatives. The original ELECTRE method is labeled as 'ELECTRE I' and the evolutions have continued with ELECTRE II, ELECTRE III, ELECTRE IV, ELECTRE IS and ELECTRE TRI . ELECTRE methods comprise two main procedures: construction of one or several outranking relation(s) and an exploitation procedure. Unlike many other MCDM methods, in the ELECTRE method, it is not assumed that the criteria are mutually independent. One of the disadvantages of the ELECTRE method is about the parameters of discordance and concordance thresholds. It is difficult for a decision maker to provide any justification for the values chosen for these parameters. The TOPSIS method, which was developed by Hwang and Yoon (Satty, 1990), is a value-based compensatory method. This method attempts to rank alternatives according to their distances from the ideal and nadir (positive-ideal and negative-ideal) solutions. However, it does not consider the relative importance of these distances (Opricovic and Tzeng, 2004). PROMETHEE is an MCDM method for ranking a finite set of alternative with respect to some conflicting criteria. PROMETHEE is applicable even when we have simple and efficient information. This method is based on the comparison of alternatives considering the deviations of them on each criterion, and uses preference functions for criteria to determine these deviations. Then the positive and negative preference flows are utilized for appraising and ranking the alternatives (Brans *et al.,* 1986). The COPRAS method is an efficient MCDM method which determines the best alternative according to a ratio based on two measures: benefit criteria performance summation and cost criteria performance summation. The applicability of this method is demonstrated in many real-word MCDM problems (Keshavarz Ghorabaee *et al.,* 2014; Hashemkhani Zolfani and Bahrami, 2014; Ecer, 2014; Stefano *et al.*, 2015). The VIKOR method was originally developed by Opricovic (Opricovic, 1998) to solve decision problems with conflicting and non-commensurable criteria (criteria with different units). The alternatives are evaluated according to all established criteria, and solution that is closest to the ideal is the best in this method. The logic of this method is similar to the TOPSIS method. However, there are some significant differences that assessed by Opricovic and Tzeng (2004). The MULTIMOORA method, which was developed by Brauers and Zavadskas (2010), is an extended version of the MOORA (multi-objective optimization by ratio analysis) method (Brauers and Zavadskas, 2006). It consists of three parts, namely the ratio system, the reference point, and the full multiplicative form. This method is efficient and has been applied to many MCDM problems and extended for different environments like fuzzy and grey environments (Baležentis *et al.*, 2012a; Stanujkic *et al.*, 2012; Baležentis and Baležentis, 2011). The ARAS method is an efficient MCDM method proposed by Zavadskas and Turskis (Zavadskas and Turskis, 2010) for evaluation of microclimate in office rooms. This method has been extended and used in many application fields in the past years (Baležentis *et al.*, 2012b; Dadelo *et al.*, 2012; Stanujkic, 2015). The EDAS method is relatively a new MCDM method which was proposed by Keshavarz Ghorabaee, Zavadskas, Olfat and Turskis (2015). The application of this method was examined in the multi-criteria in the multi-criteria inventory ABC classification. Moreover, it was demonstrated that the EDAS method has a good efficiency for dealing with multi-criteria decision-making problems.

All the above-mentioned MCDM methods have advantages and disadvantages which appraising them is not the aim of this paper. In this paper, we want to propose a new method to handle multi-criteria decision-making problems. This method is named CODAS, and has some features that have not been considered in the other MCDM methods. In the proposed method, the overall performance of an alternative is measured by the Euclidean and Taxicab distances from the negative-ideal point. The CODAS uses the Euclidean distance as the primary measure of assessment. If the Euclidean distances of two alternatives are very close to each other, the Taxicab distance is used to compare them. The degree of closeness of Euclidean distances is set by a threshold parameter. The Euclidean and Taxicab distances are measures for l^2-norm and l^1-norm indifference spaces, respectively (Yoon, 1987). Therefore, In the CODAS method, we first assess the alternatives in an l^2-norm indifference space. If the alternatives are not comparable in this space, we go to an l^1-norm indifference space. To perform this process, we should compare each pair of alternatives. In this study, we present the CODAS method in detail and illustrate the proposed method by using some numerical examples. Moreover, a comparative sensitivity analysis is done to represent the validity and stability of the proposed method. We use different sets of criteria weights and five MCDM methods (WASPAS, COPRAS, TOPSIS, VIKOR and EDAS) to perform this analysis.

The rest of this paper is organized as follows. In Section 2, a new combinative distance-based assessment (CODAS) method is presented in detail. In Section 3, we use some numerical examples to illustrate the process of the CODAS method. In Section 4, a comparative sensitivity analysis is made to demonstrate the efficiency of the proposed method. Conclusions are discussed in the last section.

2. Combinative distance-based assessment (CODAS) method

In this section, we present a new method to deal with multi-criteria decision-making problems. The proposed method is called CODAS, which stands for

***CO**mbinative **D**istance-based **AS**sessment*. In this method, the desirability of alternatives is determined by using two measures. The main and primary measure is related to the Euclidean distance of alternatives from the negative-ideal. Using this type of distance requires an l^2-norm indifference space for criteria. The secondary measure is the Taxicab distance which is related to the l^1-norm indifference space. It's clear that the alternative which has greater distances from the negative-ideal solution is more desirable. In this method, if we have two alternatives which are incomparable according to the Euclidean distance, the Taxicab distance is used as secondary measure. Although the l^2-norm indifference space is preferred in the CODAS, two types of indifference space could be considered in its process. Suppose that we have n alternatives and m criteria. The steps of the proposed method are presented as follows:

Step 1. Construct the decision-making matrix (X), shown as follows:

$$X = [x_{ij}]_{n\times m} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{bmatrix}, \tag{1}$$

where x_{ij} ($x_{ij} \geq 0$) denotes the performance value of ith alternative on jth criterion ($i \in \{1,2,\dots,n\}$ and $j \in \{1,2,\dots,m\}$).

Step 2. Calculate the normalized decision matrix. We use linear normalization of performance values as follows:

$$n_{ij} = \begin{cases} \dfrac{x_{ij}}{\max\limits_i x_{ij}} & if\ j \in N_b \\ \dfrac{\min\limits_i x_{ij}}{x_{ij}} & if\ j \in N_c \end{cases} \tag{2}$$

where N_b and N_c represent the sets of benefit and cost criteria, respectively.

Step 3. Calculate the weighted normalized decision matrix. The weighted normalized performance values are calculated as follows:

$$r_{ij} = w_j n_{ij} \tag{3}$$

where w_j ($0 < w_j < 1$) denotes the weight of jth criterion, and $\sum_{j=1}^{m} w_j = 1$.

Step 4. Determine the negative-ideal solution (point) as follows:

$$ns = [ns_j]_{1\times m} \tag{4}$$

$$ns_j = \min_i r_{ij} \tag{5}$$

Step 5. Calculate the Euclidean and Taxicab distances of alternatives from the negative-ideal solution, shown as follows:

$$E_i = \sqrt{\sum_{j=1}^{m} (r_{ij} - ns_j)^2} \tag{6}$$

$$T_i = \sum_{j=1}^{m} |r_{ij} - ns_j| \tag{7}$$

Step 6. Construct the relative assessment matrix, shown as follows:

$$Ra = [h_{ik}]_{n\times n} \tag{8}$$

$$h_{ik} = (E_i - E_k) + \big(\psi(E_i - E_k) \times (T_i - T_k)\big), \tag{9}$$

where $k \in \{1,2,\ldots,n\}$ and ψ denotes a threshold function to recognize the equality of the Euclidean distances of two alternatives, and is defined as follows:

$$\psi(x) = \begin{cases} 1 & if \quad |x| \geq \tau \\ 0 & if \quad |x| < \tau \end{cases} \tag{10}$$

In this function, τ is the threshold parameter that can be set by decision-maker. It is suggested to set this parameter at a value between 0.01 and 0.05. If the difference between Euclidean distances of two alternatives is ~~less~~ than τ, these two alternatives are also compared by the Taxicab distance. In this study, we use $\tau = 0.02$ for the calculations.

Step 7. Calculate the assessment score of each alternative, shown as follows:

$$\mathrm{H}_i = \sum_{k=1}^{n} h_{ik}, \tag{11}$$

Step 8. Rank the alternatives according to the decreasing values of assessment score (H_i). The alternative with the highest H_i is the best choice among the alternatives.

To describe the proposed method, we use a simple situation with seven alternatives and two criteria. Suppose that weighted normalized performance values (r_{ij}) have been calculated. These values are dimensionless and between 0 and 1. Figure 1 shows the position of all alternatives according to these values.

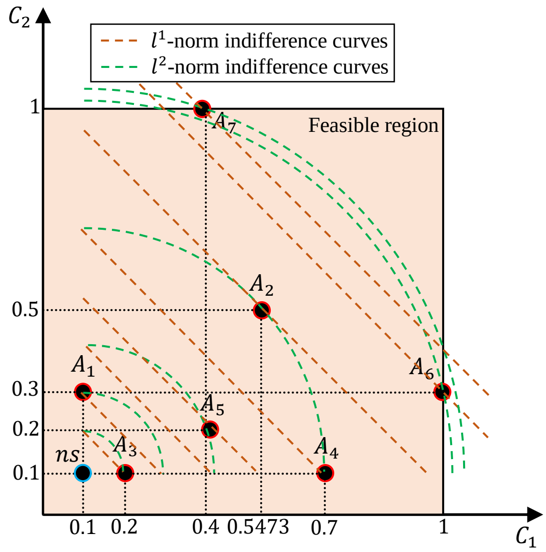

Figure 1. A simple graphical example with two criteria

As can be seen in this figure, $ns = [0.1\ 0.1]$ is the negative-ideal point (solution).The Euclidean distances of alternatives from this point are:

$$E_1 = \sqrt{(0.1 - 0.1)^2 + (0.3 - 0.1)^2} = 0.2$$

$$E_2 = \sqrt{(0.5473 - 0.1)^2 + (0.5 - 0.1)^2} = 0.6$$

$$E_3 = \sqrt{(0.2 - 0.1)^2 + (0.1 - 0.1)^2} = 0.1$$

$$E_4 = \sqrt{(0.7 - 0.1)^2 + (0.1 - 0.1)^2} = 0.6$$

$$E_5 = \sqrt{(0.4 - 0.1)^2 + (0.2 - 0.1)^2} = 0.3162$$

$$E_6 = \sqrt{(1 - 0.1)^2 + (0.3 - 0.1)^2} = 0.9220$$

$$E_7 = \sqrt{(0.4 - 0.1)^2 + (1 - 0.1)^2} = 0.9487$$

According these distances, we can say that the order of alternatives is $A_3 \prec A_1 \prec A_5 \prec A_2 = A_4 \prec A_6 \prec A_7$. As previously stated, the Euclidean distance is a measure to compare the alternatives in an l^2-norm indifference space. In this space we cannot find the difference between A_2 and A_4. So the Taxicab distance, that is the measure of l^1-norm indifference space, is used in this case. The Taxicab distances of A_2 and A_4 from the negative-ideal point are:

$$T_2 = |0.5473 - 0.1| + |0.5 - 0.1| = 0.8473$$

$$T_4 = |0.7 - 0.1| + |0.1 - 0.1| = 0.6$$

As can be seen, A_2 has greater Taxicab distance from the negative-ideal point. This fact is clear according to the indifference curves which presented in Figure 1. Therefore, we can say that A_2 is more desirable than A_4, and the final ranking is $A_3 \prec A_1 \prec A_5 \prec A_4 \prec A_2 \prec A_6 \prec A_7$.

3. Illustrative examples

To illustrate the process of the CODAS method, we use two examples in this section. The steps of the proposed method are presented through these examples.

3.1. Example 1

This example is adapted from Chakraborty and Zavadskas (2014) which is related to the selection of the most appropriate industrial robot. Five different criteria which are considered in this robot selection problem are: load capacity (in kg), maximum tip speed (in mm/s), repeatability (in mm), memory capacity (in points or steps) and manipulator reach (in mm). Among these criteria, the load capacity, maximum tip speed, memory capacity, and manipulator reach are defined as benefit criteria, and the repeatability is defined as a cost criterion. This problem consists of seven alternatives, and the corresponding data are given in Table 1.

Table 1. Data of Example 1

	Weights of criteria	0.036	0.326	0.192	0.326	0.120
Alternatives	Robots	Load capacity	Maximum tip speed	Repeatability	Memory capacity	Manipulator reach
A_1	ASEA-IRB 60/2	60	0.4	2540	500	990
A_2	Cincinnati Milacrone T3-726	6.35	0.15	1016	3000	1041
A_3	Cybotech V15 Electric Robot	6.8	0.10	1727.2	1500	1676
A_4	Hitachi America Process Robot	10	0.2	1000	2000	965
A_5	Unimation PUMA 500/600	2.5	0.10	560	500	915
A_6	United States Robots Maker 110	4.5	0.08	1016	350	508
A_7	Yaskawa Electric Motoman L3C	3	0.1	1778	1000	920

According to Table 1, we can construct the decision matrix. Then the normalized decision matrix is calculated as shown in Table 2.

Table 2. The normalized decision matrix of Example 1

Alternatives	Load capacity	Maximum tip speed	Repeatability	Memory capacity	Manipulator reach
A_1	1.000	0.200	1.000	0.167	0.591
A_2	0.106	0.533	0.400	1.000	0.621
A_3	0.113	0.800	0.680	0.500	1.000
A_4	0.167	0.400	0.394	0.667	0.576
A_5	0.042	0.800	0.220	0.167	0.546
A_6	0.075	1.000	0.400	0.117	0.303
A_7	0.050	0.800	0.700	0.333	0.549

Using weights of criteria that are given in Table 1, the weighted normalized performance values can be calculated, and then the negative-ideal solution is determined. According to the obtained values, the Euclidean and Taxicab distances of alternatives from the negative-ideal solution are also computed. The results are presented in Table 3.

Table 3. The weighted normalized decision matrix and the negative-ideal solution of Example 1

Alternatives	Load capacity	Maximum tip speed	Repeatability	Memory capacity	Manipulator reach	E_i	T_i
A_1	0.0360	0.0384	0.3260	0.0543	0.0709	0.2593	0.3394
A_2	0.0038	0.1024	0.1304	0.3260	0.0745	0.3032	0.4510
A_3	0.0041	0.1536	0.2217	0.1630	0.1200	0.2415	0.4762
A_4	0.0060	0.0768	0.1283	0.2173	0.0691	0.1947	0.3114
A_5	0.0015	0.1536	0.0719	0.0543	0.0655	0.1199	0.1606
A_6	0.0027	0.1920	0.1304	0.0380	0.0364	0.1644	0.2133
A_7	0.0018	0.1536	0.2282	0.1087	0.0659	0.2087	0.3720
Negative-ideal solution	0.0015	0.0384	0.0719	0.0380	0.0364		

The relative assessment matrix (Ra) and the assessment scores (H_i) of alternatives can be calculated by using Table 3 and Eqs. (8) to (10). Table 4 represents the results. It should be noted that, the calculations are performed with $\tau = 0.02$.

Table 4. The relative assessment matrix and the assessment scores of alternatives of Example 1

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	H_i
A_1	0.0000	-0.1554	0.0178	0.0926	0.3181	0.2210	0.0180	0.5122
A_2	0.1554	0.0000	0.0364	0.2480	0.4735	0.3764	0.1734	1.4633
A_3	-0.0178	-0.0364	0.0000	0.2116	0.4371	0.3400	0.1370	1.0715
A_4	-0.0926	-0.2480	-0.2116	0.0000	0.2255	0.1284	-0.0140	-0.2125
A_5	-0.3181	-0.4735	-0.4371	-0.2255	0.0000	-0.0971	-0.3001	-1.8515
A_6	-0.2210	-0.3764	-0.3400	-0.1284	0.0971	0.0000	-0.2030	-1.1717
A_7	-0.0180	-0.1734	-0.1370	0.0140	0.3001	0.2030	0.0000	0.1887

According to the values of assessment scores, the ranking of alternatives is $A_5 \prec A_6 \prec A_4 \prec A_7 \prec A_1 \prec A_3 \prec A_2$. Therefore, A_2 (Cincinnati Milacrone T3-726) is the best robot with respect to the assessment of the CODAS method.

3.2. Example

This example is adapted from Zavadskas and Turskis (2010) and considers the evaluation of microclimate in an office. Six criteria determined for this evaluation process are: the amount of air per head (in m^3/h), relative air humidity (in percent), air temperature (in $^{\circ}$C), illumination during work hours (in lx), rate of air flow (in m/s), and dew point (in $^{\circ}$C). All of these criteria are defined as benefit criteria except the rate of air flow and the dew point. Fourteen alternatives should be evaluated according to these criteria. The data of this problem are shown in Table 5.

Table 5. Data of Example 2

Weights of Criteria	0.21	0.16	0.26	0.17	0.12	0.08
Alternatives	The amount of air per head	Relative air humidity	Air temperature	Illumination during work hours	Rate of air flow	Dew point
A_1	7.6	46	18	390	0.1	11
A_2	5.5	32	21	360	0.05	11
A_3	5.3	32	21	290	0.05	11
A_4	5.7	37	19	270	0.05	9
A_5	4.2	38	19	240	0.1	8
A_6	4.4	38	19	260	0.1	8
A_7	3.9	42	16	270	0.1	5
A_8	7.9	44	20	400	0.05	6
A_9	8.1	44	20	380	0.05	6
A_{10}	4.5	46	18	320	0.1	7
A_{11}	5.7	48	20	320	0.05	11
A_{12}	5.2	48	20	310	0.05	11
A_{13}	7.1	49	19	280	0.1	12
A_{14}	6.9	50	16	250	0.05	10

According to steps 1 and 2 of the CODAS method and Table 5, we can construct the decision matrix and calculate the normalized performance values using Eq. (2). The normalized decision matrix is shown in Table 6. As can be seen in this table, the maximum values in benefit criteria and the minimum values of cost criteria are transformed to 1. Thus, there is no difference between the dimension (unit of measurement) and the type criteria after normalization.

Table 6. The normalized decision matrix of Example 2

Alternatives	The amount of air per head	Relative air humidity	Air temperature	Illumination during work hours	Rate of air flow	Dew point
A_1	0.938	0.920	0.857	0.975	0.500	0.455
A_2	0.679	0.640	1.000	0.900	1.000	0.455
A_3	0.654	0.640	1.000	0.725	1.000	0.455
A_4	0.704	0.740	0.905	0.675	1.000	0.556
A_5	0.519	0.760	0.905	0.600	0.500	0.625
A_6	0.543	0.760	0.905	0.650	0.500	0.625
A_7	0.481	0.840	0.762	0.675	0.500	1.000
A_8	0.975	0.880	0.952	1.000	1.000	0.833
A_9	1.000	0.880	0.952	0.950	1.000	0.833
A_{10}	0.556	0.920	0.857	0.800	0.500	0.714
A_{11}	0.704	0.960	0.952	0.800	1.000	0.455
A_{12}	0.642	0.960	0.952	0.775	1.000	0.455
A_{13}	0.877	0.980	0.905	0.700	0.500	0.417
A_{14}	0.852	1.000	0.762	0.625	1.000	0.500

To calculate the negative-ideal solution, we should obtain the weighted normalized performance values first. Table 7 shows the weighted normalized decision-matrix and corresponding negative-ideal solutions. Also, in the last two columns of this table, the Euclidean and Taxicab distances of alternatives from the negative-ideal solution are represented.

Table 7. The weighted normalized decision matrix and the negative-ideal solution of Example 2

Alternatives	The amount of air per head	Relative air humidity	Air temperature	Illumination during work hours	Rate of air flow	Dew point	E_i	T_i
A_1	0.1970	0.1472	0.2229	0.1658	0.0600	0.0364	0.1261	0.2323
A_2	0.1426	0.1024	0.2600	0.1530	0.1200	0.0364	0.1085	0.2174
A_3	0.1374	0.1024	0.2600	0.1233	0.1200	0.0364	0.0960	0.1825
A_4	0.1478	0.1184	0.2352	0.1148	0.1200	0.0444	0.0877	0.1837
A_5	0.1089	0.1216	0.2352	0.1020	0.0600	0.0500	0.0457	0.0808
A_6	0.1141	0.1216	0.2352	0.1105	0.0600	0.0500	0.0476	0.0945
A_7	0.1011	0.1344	0.1981	0.1148	0.0600	0.0800	0.0580	0.0914
A_8	0.2048	0.1408	0.2476	0.1700	0.1200	0.0667	0.1550	0.3530
A_9	0.2100	0.1408	0.2476	0.1615	0.1200	0.0667	0.1550	0.3496
A_{10}	0.1167	0.1472	0.2229	0.1360	0.0600	0.0571	0.0677	0.1429
A_{11}	0.1478	0.1536	0.2476	0.1360	0.1200	0.0364	0.1096	0.2444
A_{12}	0.1348	0.1536	0.2476	0.1318	0.1200	0.0364	0.1035	0.2272
A_{13}	0.1841	0.1568	0.2352	0.1190	0.0600	0.0333	0.1073	0.1915
A_{14}	0.1789	0.1600	0.1981	0.1063	0.1200	0.0400	0.1141	0.2063
Negative-ideal solution	0.1011	0.1024	0.1981	0.1020	0.0600	0.0333		

According to the distances given in Table 7, we can calculate the relative assessment matrix and assessment scores related to the steps 6 and 7 of the CODAS method (with $\tau = 0.02$). The results are presented in Table 8.

The calculated assessment values shows that the alternatives is prioritized as $A_5 \prec A_6 \prec A_7 \prec A_{10} \prec A_4 \prec A_3 \prec A_{13} \prec A_{12} \prec A_2 \prec A_{14} \prec A_{11} \prec A_1 \prec A_9 \prec A_8$. Therefore, we can select A_8 as the best alternative with respect to the assessment performed by the CODAS method.

Table 8. The relative assessment matrix and the assessment scores of alternatives of Example 2

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}	H_i
A_1	0.000	0.018	0.080	0.087	0.232	0.216	0.209	-0.150	-0.146	0.148	0.016	0.028	0.019	0.012	0.768
A_2	-0.018	0.000	0.012	0.054	0.199	0.184	0.176	-0.182	-0.179	0.115	-0.001	0.005	0.001	-0.006	0.363
A_3	-0.080	-0.012	0.000	0.008	0.152	0.136	0.129	-0.229	-0.226	0.068	-0.014	-0.007	-0.011	-0.018	-0.105
A_4	-0.087	-0.054	-0.008	0.000	0.145	0.129	0.122	-0.237	-0.233	0.061	-0.083	-0.016	-0.020	-0.049	-0.329
A_5	-0.232	-0.199	-0.152	-0.145	0.000	-0.002	-0.012	-0.381	-0.378	-0.084	-0.228	-0.204	-0.172	-0.194	-2.384
A_6	-0.216	-0.184	-0.136	-0.129	0.002	0.000	-0.010	-0.366	-0.363	-0.069	-0.212	-0.189	-0.157	-0.178	-2.207
A_7	-0.209	-0.176	-0.129	-0.122	0.012	0.010	0.000	-0.359	-0.355	-0.010	-0.205	-0.181	-0.149	-0.171	-2.043
A_8	0.150	0.182	0.229	0.237	0.381	0.366	0.359	0.000	0.000	0.297	0.154	0.177	0.209	0.187	2.929
A_9	0.146	0.179	0.226	0.233	0.378	0.363	0.355	0.000	0.000	0.294	0.151	0.174	0.206	0.184	2.890
A_{10}	-0.148	-0.115	-0.068	-0.061	0.084	0.069	0.010	-0.297	-0.294	0.000	-0.143	-0.120	-0.088	-0.110	-1.282
A_{11}	-0.016	0.001	0.014	0.083	0.228	0.212	0.205	-0.154	-0.151	0.143	0.000	0.006	0.002	-0.005	0.568
A_{12}	-0.028	-0.005	0.007	0.016	0.204	0.189	0.181	-0.177	-0.174	0.120	-0.006	0.000	-0.004	-0.011	0.313
A_{13}	-0.019	-0.001	0.011	0.020	0.172	0.157	0.149	-0.209	-0.206	0.088	-0.002	0.004	0.000	-0.007	0.157
A_{14}	-0.012	0.006	0.018	0.049	0.194	0.178	0.171	-0.187	-0.184	0.110	0.005	0.011	0.007	0.000	0.364

4. Comparative sensitivity analysis

To evaluate the stability and validity of the CODAS method, a comparative sensitivity analysis is performed in this section. The problem that is considered in this analysis is borrowed from Keshavarz Ghorabaee *et al.* (2015). In this problem ten alternatives are assessed on seven criteria. To make the analysis, we choose some commonly used MCDM methods for comparing the results of them with the result of the proposed method. The chosen MCDM methods include WASPAS, COPRAS, TOPSIS, VIKOR and EDAS. It should be noted that the TOPSIS method has been proposed in different versions, and we use the version that considered in the research of Opricovic and Tzeng (2004). For this comparative analysis, ten sets of criteria weights are simulated. Data of the MCDM problem and sets of criteria weights are shown in Tables 9 and 10, respectively. In the MCDM problem, C_1 to C_3 are benefit criteria, and C_4 to C_7 are cost criteria. We solve this problem using the CODAS and the selected MCDM methods in the different sets of simulated criteria weights. The results are represented in Table 11.

Table 9. Data of the MCDM problem for comparative sensitivity analysis

Alternatives	Criteria						
	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	23	264	2.37	0.05	167	8900	8.71
A_2	20	220	2.2	0.04	171	9100	8.23
A_3	17	231	1.98	0.15	192	10800	9.91
A_4	12	210	1.73	0.2	195	12300	10.21
A_5	15	243	2	0.14	187	12600	9.34
A_6	14	222	1.89	0.13	180	13200	9.22
A_7	21	262	2.43	0.06	160	10300	8.93
A_8	20	256	2.6	0.07	163	11400	8.44
A_9	19	266	2.1	0.06	157	11200	9.04
A_{10}	8	218	1.94	0.11	190	13400	10.11

Table 10. Simulated weights of criteria in different sets

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
Set 1	0.092	0.197	0.172	0.206	0.142	0.009	0.182
Set 2	0.215	0.156	0.174	0.172	0.092	0.151	0.041
Set 3	0.262	0.015	0.103	0.018	0.037	0.306	0.258
Set 4	0.086	0.258	0.011	0.118	0.105	0.207	0.215
Set 5	0.054	0.139	0.127	0.184	0.201	0.215	0.079
Set 6	0.198	0.192	0.049	0.035	0.145	0.279	0.102
Set 7	0.149	0.058	0.192	0.066	0.129	0.177	0.228
Set 8	0.303	0.174	0.044	0.047	0.082	0.268	0.082
Set 9	0.239	0.073	0.271	0.102	0.058	0.076	0.181
Set 10	0.119	0.089	0.208	0.146	0.136	0.228	0.072

Table 11. The ranking results with different methods in different sets

Set No.	Method	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
1	CODAS	2	1	7	10	6	8	3	4	5	9
	WASPAS	1	2	8	10	6	7	3	4	5	9
	COPRAS	1	2	8	10	6	7	3	4	5	9
	TOPSIS	1	3	9	10	8	7	2	4	5	6
	VIKOR	2	5	8	10	6	7	3	1	4	9
	EDAS	1	3	9	10	6	7	2	4	5	8
2	CODAS	1	2	6	9	7	8	3	4	5	10
	WASPAS	1	2	6	10	7	8	3	4	5	9
	COPRAS	1	2	6	10	7	8	3	4	5	9
	TOPSIS	1	3	6	10	8	7	2	4	5	9
	VIKOR	1	5	6	9	7	8	2	3	4	10
	EDAS	1	3	6	10	7	8	2	4	5	9
3	CODAS	1	2	6	9	7	8	3	4	5	10
	WASPAS	1	2	6	9	7	8	3	4	5	10
	COPRAS	1	2	6	9	7	8	3	4	5	10
	TOPSIS	1	2	6	9	7	8	3	4	5	10
	VIKOR	1	2	6	9	7	8	3	4	5	10
	EDAS	1	2	6	9	7	8	3	4	5	10
4	CODAS	1	2	6	10	7	8	3	5	4	9
	WASPAS	1	2	6	10	7	8	3	5	4	9
	COPRAS	1	2	6	10	7	8	3	5	4	9
	TOPSIS	1	3	7	10	8	9	2	5	4	6
	VIKOR	1	5	7	10	6	8	2	4	3	9
	EDAS	1	2	6	10	7	8	3	5	4	9
5	CODAS	2	1	6	10	7	8	3	5	4	9
	WASPAS	1	2	6	10	7	8	3	5	4	9
	COPRAS	1	2	6	10	7	8	3	5	4	9
	TOPSIS	1	2	9	10	8	7	3	5	4	6
	VIKOR	1	5	7	10	6	8	2	4	3	9
	EDAS	1	2	6	10	7	8	3	4	5	9
6	CODAS	1	2	6	9	7	8	3	4	5	10
	WASPAS	1	3	6	9	7	8	2	4	5	10
	COPRAS	1	3	6	9	7	8	2	4	5	10
	TOPSIS	1	3	6	9	7	8	2	4	5	10
	VIKOR	1	5	6	8	7	9	2	4	3	10
	EDAS	1	3	6	9	7	8	2	4	5	10
7	CODAS	1	2	6	9	7	8	4	3	5	10
	WASPAS	1	2	6	9	7	8	3	4	5	10
	COPRAS	1	2	6	10	7	8	3	4	5	9
	TOPSIS	1	3	6	10	7	8	2	4	5	9
	VIKOR	1	3	8	10	6	7	2	4	5	9
	EDAS	1	2	6	10	7	8	3	4	5	9
8	CODAS	1	2	6	9	7	8	3	4	5	10
	WASPAS	1	2	6	9	7	8	3	4	5	10
	COPRAS	1	2	6	9	7	8	3	4	5	10
	TOPSIS	1	3	6	9	7	8	2	4	5	10
	VIKOR	1	3	6	9	7	8	2	5	4	10
	EDAS	1	3	6	9	7	8	2	4	5	10
9	CODAS	1	2	6	9	7	8	3	4	5	10
	WASPAS	1	2	6	9	7	8	3	4	5	10
	COPRAS	1	2	6	10	7	8	3	4	5	9
	TOPSIS	1	4	6	10	7	8	2	3	5	9
	VIKOR	2	4	7	10	6	8	3	1	5	9
	EDAS	1	4	6	10	7	8	2	3	5	9
10	CODAS	2	1	6	10	7	8	3	4	5	9
	WASPAS	1	2	6	10	7	8	3	4	5	9
	COPRAS	1	2	6	10	7	8	3	4	5	9
	TOPSIS	1	3	7	10	9	8	2	4	5	6
	VIKOR	1	3	6	9	7	8	2	4	5	10
	EDAS	1	2	6	10	7	8	3	4	5	9

To compare the ranking results obtained from the different methods, the Spearman's rank correlation coefficient (r_s) is used. This is a suitable coefficient when we have ordinal variables or ranked variables. Table 12 represents the correlation coefficients that show the association between the results of the proposed method and the selected MCDM methods. If this correlation coefficient is greater than 0.8, the relationship between variables is very strong . As can be seen in Table 12, all values of r_s are greater than 0.8. Therefore, we can confirm the validity and stability of the results of the CODAS method.

Table 12. Correlation coefficients between the ranking results of the CODAS and the other methods

	r_s									
Method	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6	Set 7	Set 8	Set 9	Set 10
WASPAS	0.976	0.988	1	1	0.988	0.988	0.988	1	1	0.988
COPRAS	0.976	0.988	1	1	0.988	0.988	0.976	1	0.988	0.988
TOPSIS	0.855	0.964	1	0.915	0.867	0.988	0.952	0.988	0.952	0.879
VIKOR	0.830	0.927	1	0.915	0.867	0.903	0.915	0.976	0.891	0.952
EDAS	0.927	0.976	1	1	0.976	0.988	0.976	0.988	0.952	0.988

As previously mentioned, a threshold parameter (τ) is used in the process of the CODAS method. We suggest a value between 0.01 and 0.05 for this parameter. However, we want to evaluate the effect of changing this parameter on the ranking result of the CODAS methods. According to Table 12, the minimum value of the Spearman's rank correlation coefficient is in the set 1 of criteria weights ($r_s = 0.83$). So this set of criteria weights, which is more sensitive than the other sets, is selected for analysis of changing the threshold parameter. We use fifteen values for this parameter in the range of 0.01 to 1. The ranking results obtained by the CODAS method in different values of τ are presented in Table 13. The graphical changes in the ranking of alternatives are also depicted in Figure 2.

Table 13. Ranking results with different values of τ

	τ														
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.015	0.2	0.3	0.5	1
A_1	2	2	2	2	2	2	2	2	5	5	1	2	2	2	2
A_2	1	1	1	1	1	1	1	1	1	2	2	1	1	1	1
A_3	7	7	7	7	7	7	7	9	7	7	7	7	7	7	7
A_4	10	10	10	10	10	10	10	10	10	9	10	10	10	10	10
A_5	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
A_6	8	8	8	8	8	8	8	7	8	8	8	8	8	8	8
A_7	3	3	3	3	3	3	3	5	3	1	3	3	3	3	3
A_8	4	4	4	4	4	4	4	3	2	3	4	4	4	4	4
A_9	5	5	5	5	5	5	5	4	4	4	5	5	5	5	5
A_{10}	9	9	9	9	9	9	9	8	9	10	9	9	9	9	9

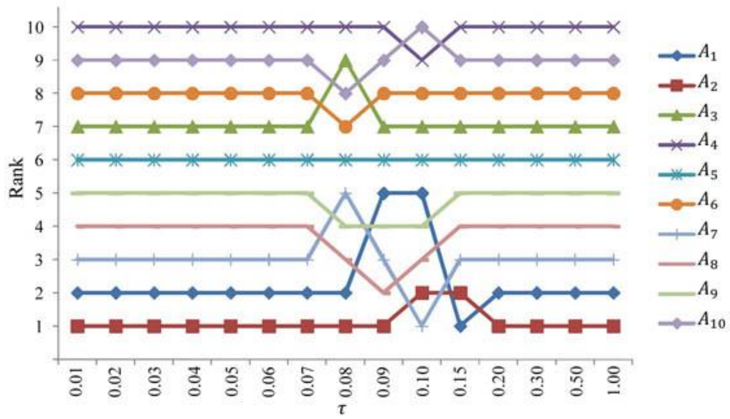

Figure 2. Effect of changing the τ parameter on ranking of alternatives

According to Table 13 and Figure 2, we can see the instability in the ranking of alternatives when the τ parameter is varied from 0.07 to 0.2. However, changing the τ parameter has not a great effect on the ranking of alternatives that can undermine the validity of the results. Therefore, we can confirm the results of the CODAS method.

5. Conclusion

Multi-criteria decision-making has increasingly been applied to many real-world problems. Many methods and techniques have also been proposed and improved by researchers in the recent years. In this paper, we have proposed a new combinative distance-based assessment (CODAS) method to handle multi-criteria decision-making problems. To assess the alternatives on multiple criteria, the proposed method uses two types of distances: Euclidean distance and Taxicab distance. These distances are calculated according to the negative-ideal solution. Therefore, the alternative which has greater distances is more desirable. However, in this process, the Euclidean distance is considered as a primary measure and the Taxicab distance is considered as a secondary measure. Two numerical examples have been used to illustrate the CODAS method. Moreover, we have performed a comparative sensitivity analysis to demonstrate the validity and stability of the proposed method. In this analysis, ten sets of criteria weights are simulated and the results of the CODAS method have been compared with the results of some existing MCDM methods. According to the results of this analysis, we can say that the proposed method is efficient to deal with MCDM problems.

REFERENCES

[1] **Baležentis, A., Baležentis, T. (2011), *An Innovative Multi-Criteria Supplier Selection Based on Two-Tuple Multimoora and Hybrid Data.*** *Economic Computation and Economic Cybernetics Studies and Research; ASE Publishing;* 2: 1-20;

[2] **Baležentis, T., Baležentis, A. (2014), *A Survey on Development and Applications of the Multi-criteria Decision Making Method MULTIMOORA.*** *Journal of Multi-Criteria Decision Analysis* 21: 209-222;

[3] **Baležentis, A., Baležentis, T., Brauers, W.K. (2012a), *MULTIMOORA-FG: A Multi-Objective Decision Making Method for Linguistic Reasoning with an Application to Personnel Selection.*** *Informatica* 23: 173-190;

[4] **Baležentis, A., Baležentis, T., Misiunas, A. (2012b), *An Integrated Assessment of Lithuanian Economic Sectors Based on Financial Ratios and Fuzzy MCDM Methods.*** *Technological and Economic Development of Economy* 18: 34-53;

[5] **Belton, V., Stewart, T. (2002), *Multiple Criteria Decision Analysis: An Integrated Approach***; *Springer US;*

[6] **Brans, J.P., Vincke, P. (1985), *Note—A Preference Ranking Organisation Method.*** *Management Science* 31: 647-656;

[7] **Brans, J.P., Vincke, P., Mareschal, B. (1986), *How to Select and How to Rank Projects: The Promethee Method.*** *European Journal of Operational Research* 24: 228-238;

[8] **Brauers, W.K.M., Zavadskas, E.K. (2006), *The MOORA Method and Its Application to Privatization in a Transition Economy.*** *Control and Cybernetics* 35: 445;

[9] **Brauers, W.K.M., Zavadskas, E.K. (2010), *Project Management by Multimoora as an Instrument for Transition Economies.*** *Technological and Economic Development of Economy* 16: 5-24;

[10] **Chakraborty, S., Zavadskas, E.K. (2014), *Applications of WASPAS Method in Manufacturing Decision Making.*** *Informatica* 25: 1-20;

[11] **Dadelo, S., Turskis, Z., Zavadskas, E.K., Dadeliene, R. (2012), *Multiple Criteria Assessment of Elite Security Personal on the Basis of ARAS and Expert Methods.*** *Economic Computation and Economic Cybernetics Studies and Research; ASE Publishing;* 46: 65-87;

[12] **Džiugaitė-Tumėnienė, R., Lapinskienė, V. (2014); *The Multicriteria Assessment Model for an Energy Supply System of a Low Energy House.*** *Engineering Structures and Technologies* 6: 33-41;

[13] **Ecer, F. (2014), *A Hybrid Banking Websites Quality Evaluation Model Using AHP and COPRAS-G: A Turkey Case.*** *Technological and Economic Development of Economy* 20: 758-782;

[14] **Fishburn, P.C. (1967), *Additive Utilities with Incomplete Product Sets: Application to Priorities and Assignments.*** *Operations Research* 15: 537-542;

[15] **Hashemkhani Zolfani, S., Bahrami, M. (2014), *Investment Prioritizing in High Tech Industries Based on SWARA-COPRAS Approach.*** *Technological and Economic Development of Economy* 20: 534-553;

[16] **Ho, W., Xu, X., Dey, P.K. (2010), *Multi-criteria Decision Making Approaches for Supplier Evaluation and Selection: A Literature Review.*** *European Journal of Operational Research* 202: 16-24;

[17] **Hwang, C.L., Yoon, K. (1981), *Multiple Attribute Decision Making: Methods and Applications : A State-of-the-Art Survey***; *Springer-Verlag;*

[18] **Jato-Espino, D., Castillo-Lopez, E., Rodriguez-Hernandez, J., Canteras-Jordana, J.C. (2014), *A Review of Application of Multi-Criteria Decision Making Methods in Construction.*** *Automation in Construction* 45: 151-162;

[19] **Keshavarz Ghorabaee, M., Zavadskas, E.K., Olfat, L., Turskis, Z. (2015), *Multi-criteria Inventory Classification Using a New Method of Evaluation Based on Distance from Average Solution (EDAS).*** *Informatica* 26: 435-451;

[20] **Kabir, G., Sadiq, R., Tesfamariam, S. (2014), *A Review of Multi-criteria Decision-making Methods for Infrastructure Management.*** *Structure and Infrastructure Engineering* 10: 1176-1210;

[21] **Keshavarz Ghorabaee, M., Amiri, M., Salehi Sadaghiani, J., Hassani Goodarzi, G. (2014), *Multiple Criteria Group Decision-making for Supplier Selection Based on COPRAS Method with Interval Type-2 Fuzzy Sets***. *The International Journal of Advanced Manufacturing Technology* 75: 1115-1130;

[22] **Miller, D.W., Starr, M.K. (1969), *Executive Decisions and Operations Research***; *Prentice-Hall*;

[23] **Opricovic, S. (1998), *Multicriteria Optimization of Civil Engineering Systems.*** *Faculty of Civil Engineering*, Belgrade;

[24] **Opricovic, S., Tzeng, G.-H. (2004), *Compromise Solution by MCDM Methods: A Comparative Analysis of VIKOR and TOPSIS.*** *European Journal of Operational Research* 156: 445-455;

[25] **Petković, D., Madić, M., Radovanović, M., Janković, P. (2015), *Application of Recently Developed MCDM Methods for Materials Selection.*** *Applied Mechanics & Materials* 809-810: 1468-1473;

[26] **Roy, B. (1968), *Classement et choix en présence de points de vue multiples***. *RAIRO-Operations Research-Recherche Opérationnelle* 2: 57-75;

[27] **Saaty, T.L. (1981), *How to Make a Decision: The Analytic Hierarchy Process.*** *European Journal of Operational Research* 48: 9-26;

[28] **Stanujkic, D. (2015),** ***Extension of the ARAS Method for Decision-Making Problems with Interval-Valued Triangular Fuzzy Numbers.*** *Informatica* 26: 335-355;

[29] **Stanujkic, D., Magdalinovic, N., Stojanovic, S., Jovanovic, R. (2012),** ***Extension of Ratio System Part of MOORA Method for Solving Decision-Making Problems with Interval Data.*** *Informatica* 23: 141-154;

[30] **Stefano, N.M., Filho, N.C., Vergara, L.G.L., Rocha, R.U.G.d. (2015),** ***COPRAS (Complex Proportional Assessment): State of the Art Research and its Applications.*** *IEEE Latin America Transactions* 13: 3899-3906;

[31] **Vafaeipour, M., Hashemkhani Zolfani, S., Morshed Varzandeh, M.H., Derakhti, A., Keshavarz Eshkalag, M. (2014),** ***Assessment of Regions Priority for Implementation of Solar Projects in Iran: New Application of a Hybrid Multi-criteria Decision Making Approach.*** *Energy Conversion and Management* 86: 653-663;

[32] **Yoon, K. (1987),** ***A Reconciliation among Discrete Compromise Solutions.*** *The Journal of the Operational Research Society* 38: 277-286;

[33] **Zavadskas, E., Kaklauskas, A. (1996),** ***Determination of an Efficient Contractor by Using the New Method of Multicriteria Assessment. International Symposium for "The Organization and Management of Construction".*** *Shaping Theory and Practice*, pp. 94-104;

[34] **Zavadskas, E.K., Turskis, Z. (2010),** ***A New Additive Ratio Assessment (ARAS) Method in Multicriteria Decision-Making.*** *Technological and Economic Development of Economy* 16: 159-172;

[35] **Zavadskas, E.K., Turskis, Z., Antucheviciene, J., Zakarevicius, A. (2012),** ***Optimization of Weighted Aggregated Sum Product Assessment.*** *Elektronika ir elektrotechnika* 122: 3-6.