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EDA$\mathcal{S}$ Method for Multi-Criteria Group Decision Making Based on Intuitionistic Fuzzy Rough Aggregation Operators

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ABSTRACT The primitive notions of rough sets and intuitionistic fuzzy set (IFS) are general mathematical tools having the ability to handle the uncertain and imprecise knowledge easily. EDA$\mathcal{S}$ (Evaluation based on distance from average solution) method has a significant role in decision making problems, especially when more conflicting criteria exist in multicriteria group decision making (MCGDM) problems. The aim of this manuscript is to present intuitionistic fuzzy rough- EDA$\mathcal{S}$ (IFR- EDA$\mathcal{S}$) method based on IF rough averaging and geometric aggregation operators. In addition, we put forward the concept of IF rough weighted averaging (IFRWA), IF rough ordered weighted averaging (IFROWA) and IF rough hybrid averaging (IFRHA) aggregation operators. Furthermore, the concepts of IF rough weighted geometric (IFRWG), IF rough ordered weighted geometric (IFROWG) and IF rough hybrid geometric (IFRHG) aggregation operators are investigated. The basic desirable characteristics of the investigated operator are given in detail. A new score and accuracy functions are defined for the proposed operators. Next, IFR-EDA$\mathcal{S}$ model for MCGDM and their stepwise algorithm are demonstrated by utilizing the proposed approach. Finally, a numerical example for the developed model is presented and a comparative study of the investigated models with some existing methods are expressed broadly which show that the investigated models are more effective and useful than the existing approaches.

INDEX TERMS Intuitionistic fuzz sets, rough sets, averaging and geometric aggregation operators, EDA$\mathcal{S}$ method, MCGDM.

I. INTRODUCTION

In this competitive environment, the complexity in decision making (DM) problems grows with the intricacy of the socio-economic environment. So, in this scenario it becomes more problematic for a single decision expert to take an accurate and an intelligent decision. In real life, it is intensively needed to fuse a group of professional experts' opinion to achieve more satisfactory and useful results by utilizing group DM models. Therefore, multicriteria group decision making (MCGDM) has the high potential and disciplined process to improve and evaluate multiple conflicting criteria in all areas of DM to get more satisfactory and feasible DM result. In DM problems, the factual information about some fact is usually unknown, and this uncertainty makes the decision process more challenging and complex. To handle this shortcoming, Zadeh [1] investigated the dominant notion of fuzzy set whish copes this kind of imprecise information accurately. Fuzzy set information is expressed by a membership grade (MG) and its membership value is restricted to [0, 1]. After the inception of this concept, it has been widely extended in different direction with both theoretical and practical aspect. Thereafter, Atanassov [2] investigated the prominent concept of intuitionistic fuzzy set (IFS) characterized by two functions MG and nonmembership grade (NonMG). For IFS, the sum of function values of MG and

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NonMG is restricted to the interval [0, 1]. From the inception of IFS, it became a hot research area for scholars and studied its hybrid structure in different direction. Xu [3] is the pioneer to investigate the concept of IF weighted averaging (IFWA) aggregation operators. The concept of IF weighted geometric (IFWG) operators are presented by Xu and Yager [4]. Ali et al. [5] developed the graphical technique for ranking score and accuracy functions. He et al. [6] investigated the concept of IF neutral averaging operators. He et al. [7] originated the notion of geometric interaction averaging operator and presented its application in DM. Zhao et al. [8] initiated the notion of generalized IFWA, generalized IFOWA, generalized IFHA operator and applied them to DM. By using the notion of Einstein norm, Wang and Liu [9], [10] presented IF Einstein weighted averaging and geometric operators. Seikh and Mandal [11] developed the notion of IF Dombi weighted averaging and geometric (IFDWA/G) operators and presented its application in DM. By using concept Hamacher t-norm and t-conorm Huang [12] developed IF Hamacher weighted averaging, ordered weighted averaging and hybrid averaging (IFHWA/ IFHOWA/ IFHHA) operators and originated their important characteristics which are investigated broadly. By applying the concept of Archimedean t-norm and t-conorm Xia et al. [13] initiated the Archimedean IFWA and Archimedean IFWG. Yang and Chen [14] generalized the concept of averaging operator to get three new types of operators such as quasi-IF ordered weighted averaging (OWA), quasi-IF Choquet order averaging and quasi-IFOWA operator based on Dempster-Shafer belief structure. Szmidt and Kacprzyk [15] proposed the concept of entropy measurement by using IF information. Hung and Yang [16] initiated the axiomatic definition of entropy based on the idea of probability of IFS. We et al. [17] generalized the notion of entropy measure to get the new similarity measure by entropy under interval valued IF information. Ye [18] developed cosine and weighted cosine similarity measures by using IF information and presented its application on medical diagnoses. Since its appearance IFS has been extensively applied by scholars and has been generalized in in various domain for solving DM problems for detail se [19]–[21].

Pawlak [22] is the pioneer who investigated the dominant notion of rough sets theory. This theory generalized the classical set theory which deals with inexact and imprecise knowledge. In recent era, research on rough set has made a great progress in both the practical uses and the theory itself. Many scholars extended the concept of rough sets in diverse ways. Dubois and Prade [23] originated the notion of fuzzy rough set by applying fuzzy relation instead of crisp binary relation. The hybrid notion of IFS and rough set play role like a bridge between these two theories, and Cornelis et al. [24] developed the combine study of IF rough set (IFRS). Zhou and Wu [25] developed constrictive and axiomatic study by applying IFR approximation operators. By using the concept of crisp and fuzzy approximation space, Zhou and Wu [26] initiated the notion of rough IFS and IFRS and presented their constrictive and axiomatic study in detail. Bustince and Burillo [27] defined the concept of IF relation. Based on the concept of two universes Zhang et al. [28] investigated the general framework of IFRS by using general IF relation. Yun and Lee [29] developed some characteristics of IFR approximation operator based on IF relation by means of topology. Different extension of IFRS are investigated. For detail see [30]–[33]. Zhang et al. [34] developed the concepts of soft rough IFS and IF soft rough set based on soft approximation and fuzzy soft approximation space. Zhang [35] proposed the generalized IFRS based on IF covering. Zhang et al. [36] extended the notion of generalized IF soft rough set based on IF soft relation. Hussain et al. [37]–[39] presented the generalized notions of IFS by using the concept of soft set and rough set under Pythagorean and Orthopair fuzzy environment. Wang and Li [40] proposed the concept of interaction power Bonferroni mean operator by using Pythagorean fuzzy information. Wang et al. [41]–[44] investigated several aggregation operators by using the concept of trapezoidal IF number and presented its application in decision making. By applying the concept of triangular IF numbers Wang et al. [45]–[50] investigated different aggregation operators and presented their applications in group decision making.

Ghorabaee et al. [51] is the pioneer who investigated the EDAS method to solve DM problems. This method has a significant role in DM problems especially when more conflict criteria exist in MCGDM problems. Similar to the classical DM methods like TOPSIS [52] (technique for order preference by similarity to ideal solution) and VIKOR [53] (the Serbian name is 'Vlse Kriterijumska Optimizacija Kompromisno Resenje' which means multi-criteria optimization and compromise solution) are the top and most popular methods which required to find out the distance from PIS and NIS. The best alternative should have had the least distance from PIS and farthest distance from NIS. Zeng and Xiao [54] proposed IFOWA weighted averaging distance with TOPSIS method and presented its basic properties. Zeng et al. [55] investigated a new score function for IF values, and applied two top most methods VIKOR and TOPSIS for the ranking of the best alternative based on the developed score and accuracy functions. Wei [56] proposed gray relation analysis (GRA) method for MADM under IF environment. However, EDAS method aims to calculate the best alternative from a series of options based on PDAS (positive distance from average solution) and NDAS (negative distance from average solution) on the basis of average solution ($A\nu S$). These two measures indicate the difference between each solution and the $A\nu S$. Therefore, the best one should have superior value of PDAS and inferior value of NDAS. Ghorabaee et al. [57] applied EDAS method by using IF information for supplier selection. Zhang et al. [58] presented the picture fuzzy weighted averaging/geometric operator under EDAS method for MCGDM. Peng and Liu [59] developed the neutrosophic soft decision approach with similarity measure based on

EDA$\mathcal{S}$ approach. Feng et al. [60] proposed the study of EDA$\mathcal{S}$ method by applying hesitant fuzzy information. Li et al [61] presented the hybrid operator and studied its application in DM by applying EDA$\mathcal{S}$ method. Liang [62] extended the study of EDA$\mathcal{S}$ method to IF environment and presented it application energy saving project. Kahraman [63] applied the EDA$\mathcal{S}$ method for site selection by using IF information. Ilieva [64] put forward the concept of EDA$\mathcal{S}$ method for group MD by using interval fuzzy information. Karasan and Kahraman [56], [66] presented EDA$\mathcal{S}$ method by interval valued neutrosophic information. Stanujkic et al. [67] used the concept of grey number to present EDA$\mathcal{S}$ method. The concept of dynamic fuzzy approach was proposed by Keshavarz-Ghorabaee [68] for MCGDM based on EDA$\mathcal{S}$ method. Stevic et al. [69] proposed EDA$\mathcal{S}$ method for DM approach by using fuzzy information. Ghorabaee [70] proposed the concept of rank reversal phenomenon and analyzed its combine study with EDA$\mathcal{S}$ and TOPSIS methods. From the best of our knowledge and above analysis up-till now no application of EDA$\mathcal{S}$ method with the hybrid study if IFS and rough sets by applying IF averaging and geometric aggregation operators is reported in IF environment. The performance of the developed IF rough EDA$\mathcal{S}$ (IFR-EDA$\mathcal{S}$) method based on IFR averaging and geometric operators is illustrated through MCGDM. Therefore, this motivates the current research to investigate averaging and geometric operators such as IFRWA, IFROWA, IFRHA, IFRWG, IFROWG and IFRHG aggregation operators by applying EDA$\mathcal{S}$ method for MCGD.

The remaining manuscript is designed as: Section II, consisting of the basic concepts which will be helpful in onward sections. In Section III, we have put forwarded the concept of IFRS with a new score and accuracy functions for IFR values (IFRV). Some basic operations for the proposed concept are given which are based on IFRV. Then in Section IV, the concept of average aggregation operators such as IFRWA, IFROWA, IFRHA aggregation operators and their desirable characteristics are investigated broadly. In Section V, the detailed study of geometric aggregation operators such as IFRWG, IFROWG, IFRHG aggregation operators and their desirable characteristics are developed broadly. Based on the developed concepts in Section VI, we have presented IFR-EDA$\mathcal{S}$ model for MCGDM and their stepwise algorithm are demonstrated by utilizing the proposed approach. Finally, in section VII, a numerical example based on EDA$\mathcal{S}$ method is presented for the selection of the best small hydro power plant (SHPP) from the different geographical sites of Pakistan. Furthermore, a comparative study of the investigated models with some existing methods are expressed broadly which show that the investigated model is more effective and useful than the existing approaches.

II. PRELIMINARIES

Here we will put forward the notions of IFS, rough sets and its fundamental operations and relations. These concepts will connect our study with upcoming sections.

Definition 1 [2]: Let $\mathcal{N}$ be a universe set and an IFS $\mathbb{I}$ on set $\mathcal{N}$ is given as

$$\mathbb{I} = \{\langle \wp, \mu_{\mathbb{I}}(\wp), \gamma_{\mathbb{I}}(\wp) \langle | \wp \in \mathcal{N}\},$$

in which $\mu_{\mathbb{I}} : \mathcal{N} \to [0,1]$ represents the MG and $\gamma_{\mathbb{I}} : \mathcal{N} \to [0,1]$ represents the NonMG of an alternative $\wp \in \mathcal{N}$ to the set $\mathbb{I}$ having the condition that $0 \leq \mu_{\mathbb{I}}(\wp) + \gamma_{\mathbb{I}}(\wp) \leq 1$. The degree of indeterminacy is given as $\pi_{\mathbb{I}}(\wp) = 1 - (\mu_{\mathbb{I}}(\wp) + \gamma_{\mathbb{I}}(\wp))$ for an object $\wp \in \mathcal{N}$. For simplicity, $\mathbb{I}(\wp) = \wp, \mu_{\mathbb{I}}(\wp), \gamma_{\mathbb{I}}(\wp)$ is represented as $\mathbb{I} = (\mu_{\mathbb{I}}, \gamma_{\mathbb{I}})$ if there is no confusion and is called IF value (IFV). The collections of all subsets (IF subsets) in $\mathcal{N}$ will be represented by $\mathcal{P}(\mathcal{N})$ ($\mathcal{IFS}(\mathcal{N})$, respectively).

Consider $\mathbb{I}_1 = (\mu_{\mathbb{I}_1}, \gamma_{\mathbb{I}_1})$ and $\mathbb{I}_2 = (\mu_{\mathbb{I}_2}, \gamma_{\mathbb{I}_2})$ are two IFVs. Then the following operations are defined on them.

(i) $\mathbb{I}_1 \cup \mathbb{I}_2 = \left(\max\left(\mu_{\mathbb{I}_1}(\wp), \mu_{\mathbb{I}_2}(\wp)\right), min\left(\left(\gamma_{\mathbb{I}_1}(\wp), \gamma_{\mathbb{I}_2}(\wp)\right)\right)\right)$;

(ii) $\mathbb{I}_1 \cap \mathbb{I}_2 = \left(\min\left(\mu_{\mathbb{I}_1}(\wp), \mu_{\mathbb{I}_2}(\wp)\right), max\left(\gamma_{\mathbb{I}_1}(\wp), \gamma_{\mathbb{I}_2}(\wp)\right)\right)$;

(iii) $\mathbb{I}_1 \oplus \mathbb{I}_2 = \left(\mu_{\mathbb{I}_1} + \mu_{\mathbb{I}_2} - \mu_{\mathbb{I}_1}\mu_{\mathbb{I}_2}, \gamma_{\mathbb{I}_1}\gamma_{\mathbb{I}_2}\right)$;

(iv) $\mathbb{I}_{r_1} \oplus \mathbb{I}_{r_2} = \left(\mu_{\mathbb{I}_1}\mu_{\mathbb{I}_2}, \gamma_{\mathbb{I}_1} + \gamma_{\mathbb{I}_2} - \gamma_{\mathbb{I}_1}\gamma_{\mathbb{I}_2}\right)$;

(v) $\mathbb{I}_1 \leq \mathbb{I}_2$ *if* $\mu_{\mathbb{I}_1}(\wp) \leq \mu_{\mathbb{I}_2}(\wp), \gamma_{\mathbb{I}_1}(\wp) \geq \gamma_{\mathbb{I}_2}(\wp)$ for all $\wp \in \mathcal{N}$;

(vi) $\mathbb{I}_1^c = \left(\gamma_{\mathbb{I}_1}, \mu_{\mathbb{I}_1}\right)$ where $\mathbb{I}_1^c$ represents the complement of $\mathbb{I}_1$;

(vii) $\alpha\mathbb{I}_1 = \left(1 - \left(1 - \mu_{\mathbb{I}_1}\right)^{\alpha}, \gamma_{\mathbb{I}_1}^{\alpha}\right)$ for $\alpha \geq 1$;

(viii) $\mathbb{I}_1^{\alpha} = \left(\mu_{\mathbb{I}_1}^{\alpha}, 1 - \left(1 - \gamma_{\mathbb{I}_1}\right)^{\alpha}\right)$ for $\alpha \geq 1$

Definition 2 [25]: Let $\mathcal{N}$ be a universal set and $\mathcal{J} \in \mathcal{N} \times \mathcal{N}$ be crisp relation. Then

(i) $\mathcal{J}$ is reflexive if $(\wp, \wp) \in \mathcal{J}, \forall \wp \in \mathcal{N}$;

(ii) $\mathcal{J}$ is symmetric if $\forall \wp, c \in \mathcal{N}, (\wp, c) \in \mathcal{J}$ then $(c, \wp) \in \mathcal{J}$;

(iii) $\mathcal{J}$ is transitive if $\forall \wp, c, d \in \mathcal{N}, (\wp, c) \in \mathcal{J}$ and $(c, d) \in \mathcal{J}$, then $(\wp, d) \in \mathcal{J}$.

Definition 3 [25]: Consider a universal set $\mathcal{N}$ and $\mathcal{J} \in \mathcal{N} \times \mathcal{N}$ be any arbitrary relation on set $\mathcal{N}$. Now defined a set valued mapping $\mathcal{J}^* : \mathcal{N} \to \mathcal{P}(\mathcal{N})$ as:

$$\mathcal{J}^*(\wp) = \{c \in \mathcal{N} | (\wp, c) \in \mathcal{J}\}, \quad for\ \wp \in \mathcal{N}$$

where $\mathcal{J}^*(\wp)$ is known as successor neighborhood of an object $\wp$ w.r.t $\mathcal{J}$. The pair $(\mathcal{N}, \mathcal{J})$ is said to be crisp approximation space. Now for any $\mathcal{K} \subseteq \mathcal{N}$, the lower and upper approximation of $\mathcal{K}$ w.r.t approximation space $(\mathcal{N}, \mathcal{J})$ is denoted and defined as:

$$\underline{\mathcal{J}}(\mathcal{K}) = \left\{\wp \in \mathcal{N} | \mathcal{J}^*(\wp) \subseteq \mathcal{K}\right\}$$
$$\overline{\mathcal{J}}(\mathcal{K}) = \left\{\wp \in \mathcal{N} | \mathcal{J}^*(\wp) \cap \mathcal{K} \neq \emptyset\right\}$$

Therefore, $\left(\underline{\mathcal{J}}(\mathcal{K}), \overline{\mathcal{J}}(\mathcal{K})\right)$ is known as rough set and $\overline{\mathcal{J}}(\mathcal{K}), \underline{\mathcal{J}}(\mathcal{K}) : \mathcal{P}(\mathcal{N}) \to \mathcal{P}(\mathcal{N})$ are upper and lower approximation operators.

Definition 4 [25]: Consider $\mathcal{N}$ is a universal set and $\mathcal{J} \in \mathcal{IFS}(\mathcal{N} \times \mathcal{N})$ be IF relation. Then

(i) $\mathcal{J}$ is reflexive if $\mu_{\mathcal{J}}(\wp, \wp) = 1$ and $\gamma_{\mathcal{J}}(\wp, \wp) = 0$, $\forall \ \wp \in \mathcal{N}$;

TABLE 1. IF relation from set $\mathcal{N}$ to $\mathcal{N}$.

$\mathcal{I}$	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$
$\wp_1$	(0.6,0.1)	(0.7,0.2)	(0.5,0.3)	(0.4,0.3)
$\wp_2$	(0.4,0.3)	(0.9,0.1)	(0.4,0.2)	(0.85,0.13)
$\wp_3$	(0.82,0.12)	(0.6,0.3)	(0.7,0.15)	(0.6,0.3)
$\wp_4$	(0.86,0.13)	(0.4,0.2)	(0.6,0.4)	(0.7,0.2)

(ii) $\mathcal{I}$ is symmetric if $\forall\ (\wp, c) \in \mathcal{N} \times \mathcal{N}$, $\mu_{\mathcal{I}}(\wp, c) = \mu_{\mathcal{I}}(c, \wp)$ and $\gamma_{\mathcal{I}}(\wp, c) = \gamma_{\mathcal{I}}(c, \wp)$;

(iii) $\mathcal{I}$ is transitive if $\forall (\wp, d) \in \mathcal{N} \times \mathcal{N}$, $\mu_{\mathcal{I}}(\wp, d) \geq \bigvee_{c\in\mathcal{N}} [\mu_{\mathcal{I}}(\wp, c) \wedge \mu_{\mathcal{I}}(c, d)]$ and $\gamma_{\mathcal{I}}(\wp, d) = \bigwedge_{c\in\mathcal{N}} [\gamma_{\mathcal{I}}(\wp, c) \wedge \gamma_{\mathcal{I}}(c, d)]$.

III. CONSTRUCTION OF INTUITIONISTIC FUZZY ROUGH SET

Here we will develop the hybrid notion of rough sets and IFS to obtained the notion of IF rough sets and initiate the new score and accuracy functions and also put forward its basic operations in detailed.

Definition 5: Consider $\mathcal{N}$ be a universal set and for any subset $\mathcal{I} \in \mathcal{IFS}(\mathcal{N} \times \mathcal{N})$ is said to be an IF relation. Then the pair $(\mathcal{N}, \mathcal{I})$ is said to be IF approximation space. Now for any $\mathcal{K} \subseteq \mathcal{IFS}(\mathcal{N})$, then the upper and lower approximations of $\mathcal{K}$ w.r.t IF approximation space $(\mathcal{N}, \mathcal{I})$ are two IFSs, which is denoted by $\bar{\mathcal{I}}(\mathcal{K})$ and $\underline{\mathcal{I}}(\mathcal{K})$ and is defined as:

$$\bar{\mathcal{I}}(\mathcal{K}) = \left\{\wp, \mu_{\bar{\mathcal{I}}(\mathcal{K})}(\wp), \gamma_{\bar{\mathcal{I}}(\mathcal{K})}(\wp) \,|\wp \in \mathcal{N}\right\}$$
$$\underline{\mathcal{I}}(\mathcal{K}) = \left\{\wp, \mu_{\underline{\mathcal{I}}(\mathcal{K})}(\wp), \gamma_{\underline{\mathcal{I}}(\mathcal{K})}(\wp) \,|\wp \in \mathcal{N}\right\},$$

where

$$\mu_{\bar{\mathcal{I}}(\mathcal{K})}(\wp) = \bigvee_{c\in\mathcal{N}} [\mu_{\mathcal{I}}(\wp, c) \vee \mu_{\mathcal{K}}(c)],$$
$$\gamma_{\bar{\mathcal{I}}(\mathcal{K})}(\wp) = \bigwedge_{c\in\mathcal{N}} [\gamma_{\mathcal{I}}(\wp, c) \wedge \gamma_{\mathcal{K}}(c)]$$
$$\mu_{\underline{\mathcal{I}}(\mathcal{K})}(\wp) = \bigwedge_{c\in\mathcal{N}} [\mu_{\mathcal{I}}(\wp, c) \wedge \mu_{\mathcal{K}}(c)],$$
$$\gamma_{\underline{\mathcal{I}}(\mathcal{K})}(\wp) = \bigvee_{c\in\mathcal{N}} [\gamma_{\mathcal{I}}(\wp, c) \vee \gamma_{\mathcal{K}}(c)]$$

such that $0 \leq \mu_{\bar{\mathcal{I}}(\mathcal{K})}(\wp) + \gamma_{\bar{\mathcal{I}}(\mathcal{K})}(\wp) \leq 1$ and $q \leq \mu_{\underline{\mathcal{I}}(\mathcal{K})}(\wp) + \gamma_{\underline{\mathcal{I}}(\mathcal{K})}(\wp) \leq 1$. As $\bar{\mathcal{I}}(\mathcal{K})$ and $\underline{\mathcal{I}}(\mathcal{K})$ are IFSs, so $\bar{\mathcal{I}}(\mathcal{K}), \underline{\mathcal{I}}(\mathcal{K}) : \mathcal{IFS}(\mathcal{N}) \to \mathcal{IFS}(\mathcal{N})$ are upper and lower approximation operators. Then the pair $\mathcal{I}(\mathcal{K}) = \left(\underline{\mathcal{I}}(\mathcal{K}), \bar{\mathcal{I}}(\mathcal{K})\right) = \left\{\wp, \left(\mu_{\underline{\mathcal{I}}(\mathcal{K})}(\wp), \gamma_{\underline{\mathcal{I}}(\mathcal{K})}(\wp)\right), \left(\mu_{\bar{\mathcal{I}}(\mathcal{K})}(\wp), \gamma_{\bar{\mathcal{I}}(\mathcal{K})}\right) |\wp \in \mathcal{K}\right\}$ is known as IF rough set. For simplicity $\mathcal{I}(\mathcal{K}) = \left\{\wp, \left(\mu_{\underline{\mathcal{I}}(\mathcal{K})}(\wp), \gamma_{\underline{\mathcal{I}}(\mathcal{K})}(\wp)\right), \left(\mu_{\bar{\mathcal{I}}(\mathcal{K})}(\wp), \gamma_{\bar{\mathcal{I}}(\mathcal{K})}\right) |\wp \in \mathcal{K}\right\}$ is represented as $\mathcal{I}(\mathcal{K}) = \left(\left(\underline{\mu}, \underline{\gamma}\right), (\overline{\mu}, \overline{\gamma})\right)$ known as IFRV.

Now to present the counter example for better explanation of the above concept of IFRS.

Example 1: Suppose $\mathcal{N} = \{\wp_1, \wp_2, \wp_3, \wp_4\}$ be an arbitrary set and $(\mathcal{N}, \mathcal{I})$ be IF approximation space with $\mathcal{I} \in \mathcal{IFS}(\mathcal{N} \times \mathcal{N})$ be IF relation as given in Table 1.

Now a decision expert presents the optimum normal decision object $\mathcal{K}$ which is an IFS, that is:

$$\mathcal{K} = \{\langle\wp_1, 0.8, 0.2\rangle, \langle\wp_2, 0.4, 0.3\rangle, \langle\wp_3, 0.5, 0.4\rangle, \langle\wp_4, 0.7, 0.25\rangle\}$$

Now to determine $\underline{\mathcal{I}}(\mathcal{K})$ and $\bar{\mathcal{I}}(\mathcal{K})$, we have

$$\begin{aligned}&\mu_{\bar{\mathcal{I}}(\mathcal{K})}(\wp_1)\\ &= \bigvee_{c\in\mathcal{N}} [\mu_{\mathcal{I}}(\wp, c) \vee \mu_{\mathcal{K}}(c)]\\ &= (0.6 \vee 0.8) \vee (0.7 \vee 0.4) \vee (0.5 \vee 0.5) \vee (0.4 \vee 0.7)\\ &= 0.8\end{aligned}$$

$$\begin{aligned}&\gamma_{\bar{\mathcal{I}}(\mathcal{K})}(\wp_1)\\ &= \bigwedge_{c\in\mathcal{N}} [\gamma_{\mathcal{I}}(\wp, c) \wedge \gamma_{\mathcal{K}}(c)]\\ &= (0.1 \wedge 0.2) \wedge (0.2 \wedge 0.3) \wedge (0.3 \wedge 0.4) \wedge (0.3 \wedge 0.25) = 0.1\end{aligned}$$

Likewise, we get the others values:

$\mu_{\bar{\mathcal{I}}(\mathcal{K})}(\wp_2) = 0.9$, $\gamma_{\bar{\mathcal{I}}(\mathcal{K})}(\wp_2) = 0.1$, $\mu_{\bar{\mathcal{I}}(\mathcal{K})}(\wp_3) = 0.82$, $\gamma_{\bar{\mathcal{I}}(\mathcal{K})}(\wp_3) = 0.15$, $\mu_{\bar{\mathcal{I}}(\mathcal{K})}(\wp_4) = 0.86$, $\gamma_{\bar{\mathcal{I}}(\mathcal{K})}(\wp_4) = 0.13$

Similarly,

$\mu_{\underline{\mathcal{I}}(\mathcal{K})}(\wp_1) = 0.4$, $\gamma_{\underline{\mathcal{I}}(\mathcal{K})}(\wp_1) = 0.4$, $\mu_{\underline{\mathcal{I}}(\mathcal{K})}(\wp_2) = 0.4$, $\gamma_{\underline{\mathcal{I}}(\mathcal{K})}(\wp_2) = 0.4$, $\mu_{\underline{\mathcal{I}}(\mathcal{K})}(\wp_3) = 0.4$, $\gamma_{\underline{\mathcal{I}}(\mathcal{K})}(\wp_3) = 0.4$, $\mu_{\underline{\mathcal{I}}(\mathcal{K})}(\wp_4) = 0.4$, $\gamma_{\underline{\mathcal{I}}(\mathcal{K})}(\wp_4) = 0.4$

Thus the upper and lower IF rough approximation operators are;

$$\bar{\mathcal{I}}(\mathcal{K}) = \left\{\begin{array}{c}\langle\wp_1, 0.8, 0.1\rangle, \langle\wp_2, 0.9, 0.1\rangle,\\ \langle\wp_3, 0.82, 0.15\rangle, \langle\wp_4, 0.86, 0.13\rangle\end{array}\right\}$$
$$\underline{\mathcal{I}}(\mathcal{K}) = \left\{\begin{array}{c}\langle\wp_1, 0.4, 0.4\rangle, \langle\wp_2, 0.4, 0.4\rangle, \langle\wp_3, 0.4, 0.4\rangle,\\ \langle\wp_4, 0.4, 0.4\rangle\end{array}\right\}$$

Therefore,

$$\begin{aligned}&\mathcal{I}(\mathcal{K})\\ &= \left(\underline{\mathcal{I}}(\mathcal{K}), \bar{\mathcal{I}}(\mathcal{K})\right)\\ &= \left\{\begin{array}{c}\langle\wp_1, (0.4, 0.4), (0.6, 0.2)\rangle, \langle\wp_2, (0.4, 0.4), (0.4, 0.1)\rangle,\\ \langle\wp_3, (0.4, 0.4), (0.6, 0.2)\rangle, \langle\wp_4, (0.4, 0.4), (0.8, 0.2)\rangle\end{array}\right\}.\end{aligned}$$

Definition 6: Let $\mathfrak{I}(\mathcal{K}_1)=\left(\underline{\mathfrak{I}}(\mathcal{K}_1),\overline{\mathfrak{I}}(\mathcal{K}_1)\right)$ and $\mathfrak{I}(\mathcal{K}_2)=\left(\underline{\mathfrak{I}}(\mathcal{K}_2),\overline{\mathfrak{I}}(\mathcal{K}_2)\right)$ be two IFRSs. Then the following operation are defined on them.

(i) $\mathfrak{I}(\mathcal{K}_1)\cup\mathfrak{I}(\mathcal{K}_2)=\left\{(\underline{\mathfrak{I}}(\mathcal{K}_1)\cup\underline{\mathfrak{I}}(\mathcal{K}_2)),\left(\overline{\mathfrak{I}}(\mathcal{K}_2)\cup\overline{\mathfrak{I}}(\mathcal{K}_2)\right)\right\}$;

(ii) $\mathfrak{I}(\mathcal{K}_1)\cap\mathfrak{I}(\mathcal{K}_2)=\left\{(\underline{\mathfrak{I}}(\mathcal{K}_1)\cap\underline{\mathfrak{I}}(\mathcal{K}_2)),\left(\overline{\mathfrak{I}}(\mathcal{K}_1)\cap\overline{\mathfrak{I}}(\mathcal{K}_2)\right)\right\}$;

(iii) $\mathfrak{I}(\mathcal{K}_1)\oplus\mathfrak{I}(\mathcal{K}_2)=\left\{(\underline{\mathfrak{I}}(\mathcal{K}_1)\oplus\underline{\mathfrak{I}}(\mathcal{K}_2)),\left(\overline{\mathfrak{I}}(\mathcal{K}_1)\oplus\overline{\mathfrak{I}}(\mathcal{K}_2)\right)\right\}$;

(iv) $\mathfrak{I}(\mathcal{K}_1)\otimes\mathfrak{I}(\mathcal{K}_2)=\left\{(\underline{\mathfrak{I}}(\mathcal{K}_1)\otimes\underline{\mathfrak{I}}(\mathcal{K}_2)),\left(\overline{\mathfrak{I}}(\mathcal{K}_1)\otimes\overline{\mathfrak{I}}(\mathcal{K}_2)\right)\right\}$;

(v) $\mathfrak{I}(\mathcal{K}_1)\subseteq\mathfrak{I}(\mathcal{K}_2)=\left(\underline{\mathfrak{I}}(\mathcal{K}_1)\subseteq\underline{\mathfrak{I}}(\mathcal{K}_2)\right)$ *and* $\left(\overline{\mathfrak{I}}(\mathcal{K}_1)\subseteq\overline{\mathfrak{I}}(\mathcal{K}_2)\right)$;

(vi) $\alpha\mathfrak{I}(\mathcal{K}_1)=\left(\alpha\underline{\mathfrak{I}}(\mathcal{K}_1),\alpha\overline{\mathfrak{I}}(\mathcal{K}_1)\right)$ for $\alpha\geq 1$;

(vii) $(\mathfrak{I}(\mathcal{K}_1))^{\alpha}=\left(\left(\underline{\mathfrak{I}}(\mathcal{K}_1)\right)^{\alpha},\left(\overline{\mathfrak{I}}(\mathcal{K}_1)\right)^{\alpha}\right)$ for $\alpha\geq 1$

(viii) $\mathfrak{I}(\mathcal{K}_1)^c=\left(\underline{\mathfrak{I}}(\mathcal{K}_1)^c,\overline{\mathfrak{I}}(\mathcal{K}_1)^c\right)$ where $\underline{\mathfrak{I}}(\mathcal{K}_1)^c$ *and* $\overline{\mathfrak{I}}(\mathcal{K}_1)^c$ represents the complement of IF rough approximation operators $\underline{\mathfrak{I}}(\mathcal{K}_1)$ *and* $\overline{\mathfrak{I}}(\mathcal{K}_1)$, i.e. $\underline{\mathfrak{I}}(\mathcal{K}_1)^c=\left(\underline{\gamma_{ij}},\underline{\mu_{ij}}\right)$

(ix) $\mathfrak{I}(\mathcal{K}_1)=\mathfrak{I}(\mathcal{K}_2)$ iff $\underline{\mathfrak{I}}(\mathcal{K}_1)=\underline{\mathfrak{I}}(\mathcal{K}_2)$ *and* $\overline{\mathfrak{I}}(\mathcal{K}_1)=\overline{\mathfrak{I}}(\mathcal{K}_2)$

To compare two or more IFRVs, we use score function for their comparison. Greater the score value of IFRV superior that value is and inferior the score value smaller that IFRV is. If the score values are equal then we use accuracy function.

Definition 7: The score function for IFRV $\mathfrak{I}(\mathcal{K})=\left(\underline{\mathfrak{I}}(\mathcal{K}),\overline{\mathfrak{I}}(\mathcal{K})\right)=\left(\left(\underline{\mu},\underline{\gamma}\right),(\overline{\mu},\overline{\gamma})\right)$ is given as:

$$\mathcal{S}(\mathfrak{I}(\mathcal{K}))=\frac{1}{4}\left(2+\underline{\mu}+\overline{\mu}-\underline{\gamma}-\overline{\gamma}\right),\quad \mathcal{S}(\mathfrak{I}(\mathcal{K}))\in[0,1].$$

Let $\mathfrak{I}(\mathcal{K})=\left(\underline{\mathfrak{I}}(\mathcal{K}),\overline{\mathfrak{I}}(\mathcal{K})\right)=\left(\left(\underline{\mu},\underline{\gamma}\right),(\overline{\mu},\overline{\gamma})\right)$ be a IFRV, then the accuracy function for $\mathfrak{I}(\mathcal{K})$ is given below:

$$\mathbb{A}c(\mathfrak{I}(\mathcal{K}))=\frac{1}{4}\left(\underline{\mu}+\overline{\mu}+\underline{\gamma}+\overline{\gamma}\right),\quad \mathbb{A}c(\mathfrak{I}(\mathcal{K}))\in[0,1].$$

Let $\mathfrak{I}(\mathcal{K}_1)=\left(\underline{\mathfrak{I}}(\mathcal{K}_1),\overline{\mathfrak{I}}(\mathcal{K}_1)\right)$ and $\mathfrak{I}(\mathcal{K}_2)=\left(\underline{\mathfrak{I}}(\mathcal{K}_2),\overline{\mathfrak{I}}(\mathcal{K}_2)\right)$ be two IFRVs. Then

(i) If $\mathcal{S}(\mathfrak{I}(\mathcal{K}_1))>S(\mathfrak{I}(\mathcal{K}_2))$, then $\mathfrak{I}(\mathcal{K}_1)>\mathfrak{I}(\mathcal{K}_2)$

(ii) If $\mathcal{S}(\mathfrak{I}(\mathcal{K}_1))<S(\mathfrak{I}(\mathcal{K}_2))$, then $\mathfrak{I}(\mathcal{K}_1)<\mathfrak{I}(\mathcal{K}_2)$

(iii) If $\mathcal{S}(\mathfrak{I}(\mathcal{K}_1))=\mathcal{S}(\mathfrak{I}(\mathcal{K}_2))$, then

  (a) If $\mathbb{A}c(\mathfrak{I}(\mathcal{K}_1))>\mathbb{A}c(\mathfrak{I}(\mathcal{K}_2))$, then $\mathfrak{I}(\mathcal{K}_1)>\mathfrak{I}(\mathcal{K}_2)$

  (b) If $\mathbb{A}c(\mathfrak{I}(\mathcal{K}_1))<\mathbb{A}c(\mathfrak{I}(\mathcal{K}_2))$, then $\mathfrak{I}(\mathcal{K}_1)<\mathfrak{I}(\mathcal{K}_2)$

  (c) If $\mathbb{A}c(\mathfrak{I}(\mathcal{K}_1))=\mathbb{A}c(\mathfrak{I}(\mathcal{K}_2))$, then $\mathfrak{I}(\mathcal{K}_1)=\mathfrak{I}(\mathcal{K}_2)$

Proposition 1: Suppose $(\mathcal{N},\mathfrak{I})$ be IF approximation space. Let $\mathfrak{I}(\mathcal{K}_1)=\left(\underline{\mathfrak{I}}(\mathcal{K}_1),\overline{\mathfrak{I}}(\mathcal{K}_1)\right)$ and $\mathfrak{I}(\mathcal{K}_2)=\left(\underline{\mathfrak{I}}(\mathcal{K}_2),\overline{\mathfrak{I}}(\mathcal{K}_2)\right)$ be any two IFRSs over $\mathcal{N}$. The following results are holds:

(i) $\sim(\sim\mathfrak{I}(\mathcal{K}_1))=\mathcal{K}_1$, where $\sim\mathfrak{I}(\mathcal{K}_1)$ is the complement of $\mathfrak{I}(\mathcal{K}_1)$;

(ii) $\mathfrak{I}(\mathcal{K}_1)\cup\mathfrak{I}(\mathcal{K}_2)=\mathfrak{I}(\mathcal{K}_2)\cup\mathfrak{I}(\mathcal{K}_1)$ *and* $\mathfrak{I}(\mathcal{K}_1)\cap\mathfrak{I}(\mathcal{K}_2)=\mathfrak{I}(\mathcal{K}_2)\cap\mathfrak{I}(\mathcal{K}_1)$

(iii) $\sim(\mathfrak{I}(\mathcal{K}_1)\cup\mathfrak{I}(\mathcal{K}_2))=(\sim\mathfrak{I}(\mathcal{K}_1))\cap(\sim\mathfrak{I}(\mathcal{K}_2))$;

(iv) $\sim(\mathfrak{I}(\mathcal{K}_1)\cap\mathfrak{I}(\mathcal{K}_2))=(\sim\mathfrak{I}(\mathcal{K}_1))\cup(\sim\mathfrak{I}(\mathcal{K}_2))$;

(v) If $\mathcal{K}_1\subseteq\mathcal{K}_2$, then $\mathfrak{I}(\mathcal{K}_1)\subseteq\mathfrak{I}(\mathcal{K}_2)$;

(vi) $\mathfrak{I}(\mathcal{K}_1\cup\mathcal{K}_2)\supseteq\mathfrak{I}(\mathcal{K}_1)\cup\mathfrak{I}(\mathcal{K}_2)$;

(vii) $\mathfrak{I}(\mathcal{K}_1\cap\mathcal{K}_2)\subseteq\mathfrak{I}(\mathcal{K}_1)\cap\mathfrak{I}(\mathcal{K}_2)$

IV. INTUITIONISTIC FUZZY ROUGH AVERAGING AGGREGATION OPERATOR

This section of manuscript consists of the notion of IF rough aggregation operators by embedding the notion of rough sets and IF averaging operators to get aggregation concepts of IFRWA, IFROWA and IFRHA operators and also presented their basic properties.

A. INTUITIONISTIC FUZZY ROUGH WEIGHTED AVERAGING OPERATOR

This subsection is devoted for the detailed study of IFRWA aggregation operators and its desirable characteristics.

Definition 8: Consider the collection $\mathfrak{I}(\mathcal{K}_i)=\left(\underline{\mathfrak{I}}(\mathcal{K}_i),\overline{\mathfrak{I}}(\mathcal{K}_i)\right)$ $(i=1,2,\ldots,n)$ of IFRVs with weight vector $\zeta=(\zeta_1,\zeta_2,\ldots,\zeta_n)^T$ such that $\sum_{i=1}^n\zeta_i=1$ and $0\leq\zeta_i\leq 1$. The IFRWA operator is determined as:

$$\text{IFRWA}(\mathfrak{I}(\mathcal{K}_1),\mathfrak{I}(\mathcal{K}_2),\ldots,\mathfrak{I}(\mathcal{K}_n))=\left(\oplus_{i=1}^n\zeta_i\underline{\mathfrak{I}}(\mathcal{K}_i),\oplus_{i=1}^n\zeta_i\overline{\mathfrak{I}}(\mathcal{K}_i)\right)$$

Based on above definition the aggregated result for IFRWA operator is illustrated in Theorem 1.

Theorem 1: Let the collection $\mathfrak{I}(\mathcal{K}_i)=\left(\underline{\mathfrak{I}}(\mathcal{K}_i),\overline{\mathfrak{I}}(\mathcal{K}_i)\right)$ $(i=1,2,\ldots,n)$ of IFRVs with weight vectors $\zeta=(\zeta_1,\zeta_2,\ldots,\zeta_n)^T$. Then IFRWA operator is determined as:

$$\begin{aligned}&\text{IFRWA}(\mathfrak{I}(\mathcal{K}_1),\mathfrak{I}(\mathcal{K}_1),\ \ldots,\mathfrak{I}(\mathcal{K}_n))\\&=\left[\oplus_{i=1}^n\zeta_i\underline{\mathfrak{I}}(\mathcal{K}_i),\oplus_{i=1}^n\zeta_i\overline{\mathfrak{I}}(\mathcal{K}_i)\right]\\&=\left[\begin{array}{c}\left(1-\prod_{i=1}^n\left(1-\underline{\mu_i}\right)^{\zeta_i},\prod_{i=1}^n\left(\underline{\gamma_i}\right)^{\zeta_i}\right),\\\left(1-\prod_{i=1}^n(1-\overline{\mu_i})^{\zeta_i},\prod_{i=1}^n(\overline{\gamma_i})^{\zeta_i}\right)\end{array}\right].\end{aligned}$$

Proof: By using mathematical induction to get the required proof.

As by defined operational law, we have

$$\begin{aligned}
&\mathcal{J}(\mathcal{K}_1) \oplus \mathcal{J}(\mathcal{K}_2)\\
&= \left[\underline{\mathcal{J}}(\mathcal{K}_1) \oplus \underline{\mathcal{J}}(\mathcal{K}_2), \overline{\mathcal{J}}(\mathcal{K}_1) \oplus \overline{\mathcal{J}}(\mathcal{K}_2)\right]\\
&= \left[\left(\underline{\mu_1} + \underline{\mu_2} - \underline{\mu_1}\,\underline{\mu_2}, \underline{\gamma_1}\,\underline{\gamma_2}\right), \left(\overline{\mu_1} + \overline{\mu_2} - \overline{\mu_1}\,\overline{\mu_2}, \overline{\gamma_1}\,\overline{\gamma_2}\right)\right]
\end{aligned}$$

and

$$\begin{aligned}
&\alpha\underline{\mathcal{J}}(\mathcal{K}_1)\\
&\quad= \left(\alpha\underline{\mathcal{J}}(\mathcal{K}_1), \alpha\overline{\mathcal{J}}(\mathcal{K}_1)\right)\\
&\quad= \left[\left(1-\left(1-\underline{\mu_1}\right)^{\alpha}, \underline{\gamma_1}^{\alpha}\right), \left(1-\left(1-\overline{\mu_1}\right)^{\alpha}, \overline{\gamma_1}^{\alpha}\right)\right]
\end{aligned}$$

Suppose $n = 2$, then

$$\begin{aligned}
&\text{IFRWA}\left(\mathcal{J}(\mathcal{K}_1), \mathcal{J}(\mathcal{K}_2)\right)\\
&\quad= \left[\oplus_{i=1}^{2}\zeta_i\underline{\mathcal{J}}(\mathcal{K}_i), \oplus_{i=1}^{2}\zeta_i\overline{\mathcal{J}}(\mathcal{K}_i)\right]\\
&\quad= \left[\begin{array}{c}\left(1-\prod_{i=1}^{2}\left(1-\underline{\mu_i}\right)^{\zeta_i}, \prod_{i=1}^{2}\underline{\gamma_i}^{\zeta_i}\right),\\ \left(1-\prod_{i=1}^{2}\left(1-\overline{\mu_i}\right)^{\zeta_i}, \prod_{i=1}^{2}\overline{\gamma_i}^{\zeta_i}\right)\end{array}\right]
\end{aligned}$$

The result is true for $n = 2$

Now suppose that result hold for $n = k$

$$\begin{aligned}
&\text{IFRWA}\left(\mathcal{J}(\mathcal{K}_1), \mathcal{J}(\mathcal{K}_2), \ldots, \mathcal{J}(\mathcal{K}_k)\right)\\
&\quad= \left[\begin{array}{c}\left(1-\prod_{i=1}^{k}\left(1-\underline{\mu_i}\right)^{\zeta_i}, \prod_{i=1}^{k}\underline{\gamma_i}^{\zeta_i}\right),\\ \left(1-\prod_{i=1}^{k}\left(1-\overline{\mu_i}\right)^{\zeta_i}, \prod_{i=1}^{k}\overline{\gamma_i}^{\zeta_i}\right)\end{array}\right]
\end{aligned}$$

Next we show that the result is true for $n = k + 1$, we have

$$\begin{aligned}
&\text{IFRWA}\left[\left(\mathcal{J}(\mathcal{K}_1), \mathcal{J}(\mathcal{K}_2), \ldots, \mathcal{J}(\mathcal{K}_k)\right), \mathcal{J}(\mathcal{K}_{k+1})\right]\\
&\quad= \left[\begin{array}{c}\left\{\left(\oplus_{i=1}^{k}\zeta_i\underline{\mathcal{J}}(\mathcal{K}_i)\right) \oplus \left(\zeta_{k+1}\underline{\mathcal{J}}(\mathcal{K}_{k+1})\right)\right\},\\ \left\{\left(\oplus_{i=1}^{k}\zeta_i\overline{\mathcal{J}}(\mathcal{K}_i)\right) \oplus \left(\zeta_{k+1}\overline{\mathcal{J}}(\mathcal{K}_{k+1})\right)\right\}\end{array}\right]\\
&\quad= \left[\begin{array}{c}\left(1-\prod_{i=1}^{k+1}\left(1-\underline{\mu_i}\right)^{\zeta_i}, \prod_{i=1}^{k+1}\underline{\gamma_i}^{\zeta_i}\right),\\ \left(1-\prod_{i=1}^{k+1}\left(1-\overline{\mu_i}\right)^{\zeta_i}, \prod_{i=1}^{k+1}\overline{\gamma_{ij}}^{\zeta_i}\right)\end{array}\right]
\end{aligned}$$

Thus the required result hold for $n = k + 1$. Hence the required result is true for all $n \geq 1$.

From the above analysis $\underline{\mathcal{J}}(\mathcal{K})$ and $\overline{\mathcal{J}}(\mathcal{K})$ are IFRVs. So, by Definition 6, $\oplus_{i=1}^{n}\zeta_i\underline{\mathcal{J}}(\mathcal{K}_i)$ and $\oplus_{i=1}^{n}\zeta_i\overline{\mathcal{J}}(\mathcal{K}_i)$ are also IFRVs. Therefore IFRWA $(\mathcal{J}(\mathcal{K}_1), \ldots, \mathcal{J}(\mathcal{K}_n))$ is also a IFRV under IF approximation space $(\mathcal{N}, \mathcal{J})$.

Example 2: Consider a set $\mathcal{K} \subseteq \mathcal{N} = \left\{\begin{array}{c}(\wp_1, \langle 0.5, 0.3\rangle, \langle 0.2, 0.1\rangle), (\wp_2, \langle 0.7, 0.1\rangle, \langle 0.4, 0.25\rangle),\\ (\wp_3, \langle 0.45, 0.13\rangle, \langle 0.8, 0.16\rangle)\end{array}\right\}$ with weight vector $\zeta = (0.32, 0.35, 0.33)^T$.

$$\begin{aligned}
&\text{IFRWA}\left(\mathcal{J}(\mathcal{K}_1), \mathcal{J}(\mathcal{K}_2), \mathcal{J}(\mathcal{K}_3)\right)\\
&= \left[\oplus_{i=1}^{3}\zeta_i\underline{\mathcal{J}}(\mathcal{K}_i), \oplus_{i=1}^{3}\zeta_i\overline{\mathcal{J}}(\mathcal{K}_i)\right]\\
&= \left[\begin{array}{c}\left\{\begin{array}{c}\left(1-(1-0.5)^{0.32}(1-0.7)^{0.35}(1-0.45)^{0.33}\right),\\ \left(0.3^{0.32} \times 0.1^{0.35} \times 0.13^{0.33}\right)\end{array}\right\},\\ \left\{\begin{array}{c}\left(1-(1-0.5)^{0.32}(1-0.7)^{0.35}(1-0.45)^{0.33}\right),\\ \left(0.3^{0.32} \times 0.1^{0.35} \times 0.13^{0.33}\right)\end{array}\right\}\end{array}\right]\\
&= \left[\langle 0.568498, 0.154982\rangle, \langle 0.542194, 0.160931\rangle\right].
\end{aligned}$$

Some important properties of IFRWA operator is initiated in Theorem 2.

Theorem 2: Consider the collection $\mathcal{J}(\mathcal{K}_i) = \left(\underline{\mathcal{J}}(\mathcal{K}_i), \overline{\mathcal{J}}(\mathcal{K}_i)\right)$ $(i = 1, 2, \ldots, n)$ of IFRVs with weight vectors $\zeta = (\zeta_1, \zeta_2, \ldots, \zeta_n)^T$ with $\sum_{i=1}^{n}\zeta_i = 1$ and $0 \leq \zeta_i \leq 1$. Then some important properties of IFRWA operator are described as:

(i) **(Idempotency):** If $\mathcal{J}(\mathcal{K}_i) = \mathcal{P}(\mathcal{K})$ for all $i = 1, 2, \ldots, n$ where $\mathcal{P}(\mathcal{K}) = \left(\underline{\mathcal{P}}(\mathcal{K}), \overline{\mathcal{P}}(\mathcal{K})\right) = \left(\left(\underline{d}, \underline{e}\right), \left(\overline{d}, \overline{e}\right)\right)$. Then *IFRWA* $(\mathcal{J}(\mathcal{K}_1), \mathcal{J}(\mathcal{K}_2), \ldots, \mathcal{J}(\mathcal{K}_n)) = \mathcal{P}(\mathcal{K})$.

(ii) **(Boundedness):** Let $(\mathcal{J}(\mathcal{K}))^- = \left(\min_i \underline{\mathcal{J}}(\mathcal{K}_i), \max_i \overline{\mathcal{J}}(\mathcal{K}_i)\right)$ and $(\mathcal{J}(\mathcal{K}))^+ = \left(\max_i \underline{\mathcal{J}}(\mathcal{K}_i), \min_j \overline{\mathcal{J}}(\mathcal{K}_i)\right)$. Then $(\mathcal{J}(\mathcal{K}))^- \leq$ *IFRWA* $(\mathcal{J}(\mathcal{K}_1), \mathcal{J}(\mathcal{K}_2), \ldots, \mathcal{J}(\mathcal{K}_n)) \leq (\mathcal{J}(\mathcal{K}_i))^+$.

(iii) **(Monotonicity):** Let $\mathcal{P}(\mathcal{L}_i) = \left(\underline{\mathcal{P}}(\mathcal{L}_i), \overline{\mathcal{P}}(\mathcal{L}_i)\right)$ $(i = 1, 2, \ldots, n)$ be another collection of IFRVs such that $\underline{\mathcal{P}}(\mathcal{L}_i) \leq \underline{\mathcal{J}}(\mathcal{K}_i)$ and $\overline{\mathcal{P}}(\mathcal{L}_i) \leq \overline{\mathcal{J}}(\mathcal{K}_i)$. Then *IFRWA* $(\mathcal{P}(\mathcal{L}_1), \mathcal{P}(\mathcal{L}_2), \ldots, \mathcal{P}(\mathcal{L}_n)) \leq$ *IFRWA* $(\mathcal{J}(\mathcal{K}_1), \mathcal{J}(\mathcal{K}_2), \ldots, \mathcal{J}(\mathcal{K}_n))$.

(iv) **(Shift invariance):** Consider another IFRV $\mathcal{P}(\mathcal{L}) = \left(\underline{\mathcal{P}}(\mathcal{L}), \overline{\mathcal{P}}(\mathcal{L})\right) = \left(\left(\underline{d}, \underline{e}\right), \left(\overline{d}, \overline{e}\right)\right)$. Then *IFRWA* $\left(\begin{array}{c}\mathcal{J}(\mathcal{K}_1) \oplus \mathcal{P}(\mathcal{L}), \mathcal{J}(\mathcal{K}_2) \oplus \mathcal{P}(\mathcal{L}), \ldots,\\ \mathcal{J}(\mathcal{K}_n) \oplus \mathcal{P}(\mathcal{L})\end{array}\right) =$ *IFRWA* $(\mathcal{J}(\mathcal{K}_1), \mathcal{J}(\mathcal{K}_2), \ldots, \mathcal{J}(\mathcal{K}_n)) \oplus \mathcal{P}(\mathcal{L})$.

(v) **(Homogeneity):** For any real number $\lambda > 0$; *IFRWA* $(\lambda\mathcal{J}(\mathcal{K}_1), \lambda\mathcal{J}(\mathcal{K}_2), \ldots, \lambda\mathcal{J}(\mathcal{K}_n)) = \lambda$*IFRWA* $(\mathcal{J}(\mathcal{K}_1), \mathcal{J}(\mathcal{K}_2), \ldots, \mathcal{J}(\mathcal{K}_n))$.

(vi) **(Commutativity):** Let $\mathcal{J}'(\mathcal{K}_i) = \left(\underline{\mathcal{J}'}(\mathcal{K}_i), \overline{\mathcal{J}'}(\mathcal{K}_i)\right)$ $(i = 1, 2, \ldots, n)$ be any permutation of $\mathcal{J}(\mathcal{K}_i) = \left(\underline{\mathcal{J}}(\mathcal{K}_i), \overline{\mathcal{J}}(\mathcal{K}_i)\right)$. Then *IFRWA* $(\mathcal{J}(\mathcal{K}_1), \mathcal{J}(\mathcal{K}_2), \ldots, \mathcal{J}(\mathcal{K}_n))$ $=$ *IFRWA* $\left(\mathcal{J}'(\mathcal{K}_1), \mathcal{J}'(\mathcal{K}_2), \ldots, \mathcal{J}'(\mathcal{K}_n)\right)$.

Proof: (*i*) **(Idempotency)** As $\mathcal{J}(\mathcal{K}_i) = \mathcal{P}(\mathcal{K})$ (for all $i = 1, 2, \ldots, n$), where $\mathcal{P}(\mathcal{K}) = \left(\underline{\mathcal{P}}(\mathcal{K}), \overline{\mathcal{P}}(\mathcal{K})\right) = \left((\underline{d}, \underline{e}), (\overline{d}, \overline{e})\right)$

$$\begin{aligned}
&\text{IFRWA}\left(\mathcal{J}(\mathcal{K}_1), \ \ldots, \mathcal{J}(\mathcal{K}_n)\right) \\
&= \left[\oplus_{i=1}^{n} \zeta_i \underline{\mathcal{J}}(\mathcal{K}_i), \oplus_{i=1}^{n} \zeta_i \overline{\mathcal{J}}(\mathcal{K}_i)\right] \\
&= \left[\begin{array}{c} \left(1-\prod_{i=1}^{k}\left(1-\underline{\mu_i}\right)^{\zeta_i}, \prod_{i=1}^{k}\underline{\gamma_i}^{\zeta_i}\right), \\ \left(1-\prod_{i=1}^{k}\left(1-\overline{\mu_i}\right)^{\zeta_i}, \prod_{i=1}^{k}\overline{\gamma_i}^{\zeta_i}\right) \end{array}\right]
\end{aligned}$$

For all $i, \mathcal{J}(\mathcal{K}_i) = \mathcal{P}(\mathcal{K}) = \left(\underline{\mathcal{P}}(\mathcal{K}), \overline{\mathcal{P}}(\mathcal{K})\right) = \left((\underline{d}, \underline{e}), (\overline{d}, \overline{e})\right)$. Therefore,

$$\begin{aligned}
&= \left[\begin{array}{c} \left(1-\prod_{i=1}^{k}\left(1-\underline{d}\right)^{\zeta_i}, \prod_{i=1}^{k}\underline{e}^{\zeta_i}\right), \\ \left(1-\prod_{i=1}^{k}\left(1-\overline{d}\right)^{\zeta_i}, \prod_{i=1}^{k}\overline{e}^{\zeta_i}\right) \end{array}\right] \\
&= \left[\left(1-\left(1-\underline{d}\right), \underline{e}\right), \left(1-\left(1-\overline{d}\right), \overline{e}\right)\right] \\
&= \left(\underline{\mathcal{P}}(\mathcal{K}), \overline{\mathcal{P}}(\mathcal{K})\right) = \mathcal{P}(\mathcal{K})
\end{aligned}$$

Hence,

$$\text{IFRWA}\left(\mathcal{J}(\mathcal{K}_1), \ \ldots, \mathcal{J}(\mathcal{K}_n)\right) = \mathcal{P}(\mathcal{K})$$

(*ii*) **(Boundedness)** As $\left(\underline{\mathcal{J}}(\mathcal{K})\right)^{-} = \left[\left(\min_i\left\{\underline{\mu_i}\right\}, \max_i\left\{\underline{\gamma_i}\right\}\right), \left(\min_i\left\{\overline{\mu_i}\right\}, \max_i\left\{\overline{\gamma_i}\right\}\right)\right]$ $\left(\underline{\mathcal{J}}(\mathcal{K})\right)^{+} = \left[\left(\max_i\left\{\underline{\mu_i}\right\}, \min_i\left\{\underline{\gamma_i}\right\}\right), \left(\max_i\left\{\overline{\mu_i}\right\}, \min_i\left\{\overline{\gamma_i}\right\}\right)\right]$ and $\mathcal{J}(\mathcal{K}_i) = \left[\left(\underline{\mu_i}, \underline{\gamma_i}\right), \left(\overline{\mu_i}, \overline{\gamma_i}\right)\right]$. To prove that

$$\begin{aligned}
\left(\mathcal{J}(\mathcal{K})\right)^{-} &\le IFRWA\left(\mathcal{J}(\mathcal{K}_1), \mathcal{J}(\mathcal{K}_2), \ldots, \mathcal{J}(\mathcal{K}_n)\right) \\
&\le \left(\mathcal{J}(\mathcal{K})\right)^{+}
\end{aligned}$$

Since for each $i = 1, 2, \ldots, n$, we have

$$\begin{aligned}
\min_i\left\{\underline{\mu_i}\right\} &\le \underline{\mu_i} \le \max_i\left\{\underline{\mu_i}\right\} \Leftrightarrow 1-\max_i\left\{\underline{\mu_i}\right\} \\
&\le 1-\underline{\mu_i} \le 1-\min_i\left\{\underline{\mu_i}\right\} \\
&\Leftrightarrow \prod_{i=1}^{n}\left(1-\max_i\left\{\underline{\mu_i}\right\}\right)^{\zeta_i} \le \prod_{i=1}^{n}\left(1-\underline{\mu_i}\right)^{\zeta_i} \\
&\le \prod_{i=1}^{n}\left(1-\min_i\left\{\underline{\mu_i}\right\}\right)^{\zeta_i} \\
&\Leftrightarrow \left(1-\max_i\left\{\underline{\mu_i}\right\}\right) \le \prod_{i=1}^{n}\left(1-\underline{\mu_i}\right)^{\zeta_i} \\
&\le \left(1-\min_i\left\{\underline{\mu_i}\right\}\right) \\
&\Leftrightarrow 1-\left(1-\min_i\left\{\underline{\mu_i}\right\}\right) \le 1-\prod_{i=1}^{n}\left(1-\underline{\mu_i}\right)^{\zeta_i} \\
&\le 1-\left(1-\max_i\left\{\underline{\mu_i}\right\}\right)
\end{aligned}$$

Hence

$$\min_i\left\{\underline{\mu_i}\right\} \le 1-\prod_{i=1}^{n}\left(1-\underline{\mu_i}\right)^{\zeta_i} \le \max_i\left\{\underline{\mu_i}\right\} \tag{1}$$

Next for each $i = 1, 2, \ldots, n$, we have

$$\begin{aligned}
\min_i\left\{\underline{\gamma_i}\right\} &\le \underline{\gamma_i} \le \max_i\left\{\underline{\gamma_i}\right\} \Leftrightarrow \prod_{i=1}^{n}\left(\min_i\left\{\underline{\gamma_i}\right\}\right)^{\zeta_i} \\
&\le \prod_{i=1}^{n}\left(\underline{\gamma_i}\right)^{\zeta_i} \le \prod_{i=1}^{n}\left(\max_i\left\{\underline{\gamma_i}\right\}\right)^{\zeta_i}
\end{aligned}$$

this implies that

$$\min_i\left\{\underline{\gamma_i}\right\} \le \prod_{i=1}^{n}\left(\underline{\gamma_i}\right)^{\zeta_i} \le \max_i\left\{\underline{\gamma_i}\right\} \tag{2}$$

Similarly, we can show that

$$\min_i\left\{\overline{\mu_i}\right\} \le 1-\prod_{i=1}^{n}\left(1-\overline{\mu_i}\right)^{\zeta_i} \le \max_i\left\{\overline{\mu_i}\right\} \tag{3}$$

and

$$\min_i\left\{\overline{\gamma_i}\right\} \le \prod_{i=1}^{n}\left(\overline{\gamma_i}\right)^{t_i} \le \max_i\left\{\overline{\gamma_i}\right\} \tag{4}$$

So from *Eqs*. (1), (2), (3) *and* (4) we have

$$\begin{aligned}
\left(\mathcal{J}(\mathcal{K})\right)^{-} &\le IFRWA\left(\mathcal{J}(\mathcal{K}_1), \mathcal{J}(\mathcal{K}_2), \ldots, \mathcal{J}(\mathcal{K}_n)\right) \\
&\le \left(\mathcal{J}(\mathcal{K}_i)\right)^{+}.
\end{aligned}$$

(*iii*) **Monotonicity:** Since $\mathcal{P}(\mathcal{L}_i) = \left(\underline{\mathcal{P}}(\mathcal{L}_i), \overline{\mathcal{P}}(\mathcal{L}_i)\right) = \left((\underline{d_i}, \underline{e_i}), (\overline{d_i}, \overline{e_i})\right)$ and $\mathcal{J}(\mathcal{K}_i) = \left(\underline{\mathcal{J}}(\mathcal{K}_i), \overline{\mathcal{J}}(\mathcal{K}_i)\right)$ To show that $\underline{\mathcal{P}}(\mathcal{L}_i) \le \underline{\mathcal{J}}(\mathcal{K}_i)$ and $\overline{\mathcal{P}}(\mathcal{L}_i) \le \overline{\mathcal{J}}(\mathcal{K}_i)$ (*for* $i = 1, 2, \ldots, n$), so

$$\begin{aligned}
&\underline{d_i} \le \underline{\mu_i} \Rightarrow 1-\underline{\mu_i} \le 1-\underline{d_i} \Rightarrow \prod_{i=1}^{n}\left(1-\underline{\mu_i}\right)^{\zeta_i} \\
&\le \prod_{i=1}^{n}\left(1-\underline{d_i}\right)^{\zeta_i} \Rightarrow 1-\prod_{i=1}^{n}\left(1-\underline{d_i}\right)^{\zeta_i} \\
&\le 1-\prod_{i=1}^{n}\left(1-\underline{\mu_i}\right)^{\zeta_i}
\end{aligned} \tag{5}$$

Next

$$\underline{e_i} \ge \underline{\gamma_i} \Rightarrow \prod_{i=1}^{n}\underline{e_i}^{\zeta_i} \ge \prod_{i=1}^{n}\underline{\gamma_i}^{\zeta_i} \tag{6}$$

Similarly, we can show that

$$1-\prod_{i=1}^{n}\left(1-\overline{d_i}\right)^{\zeta_i} \le 1-\prod_{i=1}^{n}\left(1-\overline{\mu_i}\right)^{\zeta_i} \tag{7}$$

$$\prod_{i=1}^{n}\left(\overline{d_{ij}}\right)^{\zeta_i} \geq \prod_{i=1}^{n}\left(\overline{\gamma_{ij}}\right)^{\zeta_i} \quad (8)$$

Hence from Eqs. (5), (6), (7) *and* (8), we get

$$\underline{\mathcal{P}}(\mathcal{L}_i) \leq \underline{\mathcal{I}}(\mathcal{K}_i) \quad \text{and } \overline{\mathcal{P}}(\mathcal{L}_i) \leq \overline{\mathcal{I}}(\mathcal{K}_i)$$

Therefore,

$$IFRWA(\mathcal{P}(\mathcal{L}_1), \mathcal{P}(\mathcal{L}_2), \ldots, \mathcal{P}(\mathcal{L}_n)) \leq IFRWA(\mathcal{I}(\mathcal{K}_1), \mathcal{I}(\mathcal{K}_2), \ldots, \mathcal{I}(\mathcal{K}_n))$$

(*iv*): (***Shift Invariance***) As $\mathcal{P}(\mathcal{L}) = \left(\underline{\mathcal{P}}(\mathcal{L}), \overline{\mathcal{P}}(\mathcal{L})\right) = \left((\underline{d}, \underline{e}), (\overline{d}, \overline{e})\right)$ is any IFRV and $\mathcal{I}(\mathcal{K}_i) = \left(\underline{\mathcal{I}}(\mathcal{K}_i), \overline{\mathcal{I}}(\mathcal{K}_i)\right) = \left(\left(\underline{\mu_i}, \underline{\gamma_i}\right), (\overline{\mu_i}, \overline{\gamma_i})\right)$ are the collection of IFRVs, so

$$\mathcal{I}(\mathcal{K}_1) \oplus \mathcal{P}(\mathcal{L}) = \left[\underline{\mathcal{I}}(\mathcal{K}_1) \oplus \underline{\mathcal{P}}(\mathcal{L}), \overline{\mathcal{I}}(\mathcal{K}_i) \oplus \overline{\mathcal{P}}(\mathcal{L})\right]$$

As

$$= \left(\left(1-\left(1-\underline{\mu_1}\right)(1-\underline{d}), \underline{\gamma_1}\,\underline{e}\right), (1-(1-\overline{\mu_1})(1-\overline{e}), \overline{\gamma_1}\,\overline{e})\right)$$

Therefore,

$$\begin{aligned}
&\text{IFRWA}(\mathcal{I}(\mathcal{K}_1)\oplus\mathcal{P}(\mathcal{L}), \mathcal{I}(\mathcal{K}_2)\oplus\mathcal{P}(\mathcal{L}), \ldots, \mathcal{I}(\mathcal{K}_n)\oplus\mathcal{P}(\mathcal{L}))\\
&= \left[\oplus_{i=1}^{n}\zeta_i\left(\underline{\mathcal{I}}(\mathcal{K}_i)\oplus\mathcal{P}(\mathcal{K})\right), \oplus_{i=1}^{n}\zeta_i\left(\overline{\mathcal{I}}(\mathcal{K}_i)\oplus\mathcal{P}(\mathcal{K})\right)\right]\\
&= \left[\begin{array}{c}\left(1-\prod_{i=1}^{n}\left(1-\underline{\mu_i}\right)^{\zeta_i}(1-\underline{d})^{\zeta_i}, \prod_{i=1}^{n}\underline{\gamma_i}^{\zeta_i}\underline{e}^{\zeta_i}\right),\\ \left(1-\prod_{i=1}^{n}(1-\overline{\mu_i})^{\zeta_i}\left(1-\overline{d}\right)^{\zeta_i}, \prod_{i=1}^{n}\overline{\gamma_i}^{\zeta_i}\overline{e}^{\zeta_i}\right)\end{array}\right]\\
&= \left[\begin{array}{c}\left(1-(1-\underline{d})\prod_{i=1}^{n}\left(1-\underline{\mu_i}\right)^{\zeta_i}, \underline{e}\prod_{i=1}^{n}\underline{\gamma_i}^{\zeta_i}\right),\\ \left(1-(1-\overline{d})\prod_{i=1}^{n}(1-\overline{\mu_i})^{\zeta_i}, \overline{e}\prod_{i=1}^{n}\overline{\gamma_i}^{\zeta_i}\right)\end{array}\right]\\
&= \left[\begin{array}{c}\left\{\left(1-\prod_{i=1}^{n}\left(1-\underline{\mu_i}\right)^{\zeta_i}, \prod_{i=1}^{n}\underline{\gamma_i}^{\zeta_i}\right)\oplus\left(\underline{d}, \underline{e}\right)\right\},\\ \left\{\left(1-\prod_{i=1}^{n}(1-\overline{\mu_i})^{\zeta_i}, \prod_{i=1}^{n}\overline{\gamma_i}^{\zeta_i}\right)\oplus\left(\overline{d}, \overline{e}\right)\right\}\end{array}\right]\\
&= \left[\begin{array}{c}\left(1-\prod_{i=1}^{n}\left(1-\underline{\mu_i}\right)^{\zeta_i}, \prod_{i=1}^{n}\underline{\gamma_i}^{\zeta_i}\right),\\ \left(1-\prod_{i=1}^{n}(1-\overline{\mu_i})^{\zeta_i}, \prod_{i=1}^{n}\overline{\gamma_i}^{\zeta_i}\right)\end{array}\right]\oplus\left[\left(\underline{d}, \underline{e}\right), \left(\overline{d}, \overline{e}\right)\right]\\
&= \text{IFRWA}(\mathcal{I}(\mathcal{K}_1), \mathcal{I}(\mathcal{K}_2), \ldots, \mathcal{I}(\mathcal{K}_n)) \oplus \mathcal{P}(\mathcal{L})
\end{aligned}$$

Therefore, proved is completed.

(*v*): (***Homogeneity***) For a real number $\lambda > 0$ and $\mathcal{I}(\mathcal{K}_i) = \left(\underline{\mathcal{I}}(\mathcal{K}_i), \overline{\mathcal{I}}(\mathcal{K}_i)\right)$ be a IFRVs.

As

$$\begin{aligned}
\lambda\mathcal{I}(\mathcal{K}_i) &= \left(\lambda\underline{\mathcal{I}}(\mathcal{K}_i), \lambda\overline{\mathcal{I}}(\mathcal{K}_i)\right)\\
&= \left[\left(1-\left(1-\underline{\mu_1}\right)^{\lambda}, \underline{\gamma_1}^{\lambda}\right), \left(1-(1-\overline{\mu_1})^{\lambda}, \overline{\gamma_1}^{\lambda}\right)\right]
\end{aligned}$$

Now

$$\begin{aligned}
&\text{IFRWA}(\lambda\mathcal{I}(\mathcal{K}_1), \lambda\mathcal{I}(\mathcal{K}_2), \ldots, \lambda\mathcal{I}(\mathcal{K}_n))\\
&= \left[\begin{array}{c}\left(1-\prod_{i=1}^{n}\left(1-\underline{\mu_i}\right)^{\lambda\zeta_i}, \prod_{i=1}^{n}\underline{\gamma_i}^{\lambda\zeta_i}\right),\\ \left(1-\prod_{i=1}^{n}(1-\overline{\mu_i})^{\lambda\zeta_i}, \prod_{i=1}^{n}\overline{\gamma_i}^{\lambda\zeta_i}\right)\end{array}\right]\\
&= \left[\begin{array}{c}\left\{1-\left(\prod_{i=1}^{n}\left(1-\underline{\mu_i}\right)^{\zeta_i}\right)^{\lambda}, \left(\prod_{i=1}^{n}\underline{\gamma_i}^{\zeta_i}\right)^{\lambda}\right\},\\ \left\{1-\left(\prod_{i=1}^{n}(1-\overline{\mu_i})^{\zeta_i}\right)^{\lambda}, \left(\prod_{i=1}^{n}\overline{\gamma_i}^{\zeta_i}\right)^{\lambda}\right\}\end{array}\right]\\
&= \lambda\text{IFRWA}(\mathcal{I}(\mathcal{K}_1), \mathcal{I}(\mathcal{K}_2), \ldots, \mathcal{I}(\mathcal{K}_n))
\end{aligned}$$

Hence, we get the required proof

(*vi*). Let

$$\begin{aligned}
&\text{IFRWA}(\mathcal{I}(\mathcal{K}_1), \mathcal{I}(\mathcal{K}_1), \ldots, \mathcal{I}(\mathcal{K}_n))\\
&= \left[\oplus_{i=1}^{n}\zeta_i\underline{\mathcal{I}}(\mathcal{K}_i), \oplus_{i=1}^{n}\zeta_i\overline{\mathcal{I}}(\mathcal{K}_i)\right]\\
&= \left[\begin{array}{c}\left(1-\prod_{i=1}^{n}\left(1-\underline{\mu_i}\right)^{\zeta_i}, \prod_{i=1}^{n}\left(\underline{\gamma_i}\right)^{\zeta_i}\right),\\ \left(1-\prod_{i=1}^{n}(1-\overline{\mu_i})^{\zeta_i}, \prod_{i=1}^{n}(\overline{\gamma_i})^{\zeta_i}\right)\end{array}\right],
\end{aligned}$$

Since $\left(\mathcal{I}'(\mathcal{K}_1), \mathcal{I}'(\mathcal{K}_2), \ldots, \mathcal{I}'(\mathcal{K}_n)\right)$ is any permutation of $(\mathcal{I}(\mathcal{K}_1), \mathcal{I}(\mathcal{K}_2), \ldots, \mathcal{I}(\mathcal{K}_n))$, then we have $\mathcal{I}(\mathcal{K}_i) = \mathcal{I}'(\mathcal{K}_i)$ $(i = 1, 2, \ldots, n)$

$$\begin{aligned}
&= \left[\begin{array}{c}\left(1-\prod_{i=1}^{n}\left(1-\underline{\mu_i'}\right)^{\zeta_i}, \prod_{i=1}^{n}\left(\underline{\gamma_i'}\right)^{\zeta_i}\right),\\ \left(1-\prod_{i=1}^{n}\left(1-\overline{\mu_i'}\right)^{\zeta_i}, \prod_{i=1}^{n}\left(\overline{\gamma_i'}\right)^{\zeta_i}\right)\end{array}\right]\\
&= \left[\oplus_{i=1}^{n}\zeta_i\underline{\mathcal{I}'}(\mathcal{K}_i), \oplus_{i=1}^{n}\zeta_i\overline{\mathcal{I}'}(\mathcal{K}_i)\right]\\
&= IFRWA\left(\mathcal{I}'(\mathcal{K}_1), \mathcal{I}'(\mathcal{K}_2), \ldots, \mathcal{I}'(\mathcal{K}_n)\right).
\end{aligned}$$

B. INTUITIONISTIC FUZZY ROUGH ORDERED WEIGHTED AVERAGING OPERATOR

In this subsection we put forward the concept of IFROWA operator and proposed its fundamental properties of the developed operators.

Definition 9: Consider the collection $\mathcal{I}(\mathcal{K}_i) = \left(\underline{\mathcal{I}}(\mathcal{K}_i), \overline{\mathcal{I}}(\mathcal{K}_i)\right)$ $(i = 1, 2, \ldots, n)$ of IFRVs with weight vector $\zeta = (\zeta_1, \zeta_2, \ldots, \zeta_n)^T$ such that $\sum_{i=1}^{n}\zeta_i = 1$ and $0 \leq \zeta_i \leq 1$. The IFROWA operator is determined as:

$$\text{IFROWA}(\mathcal{I}(\mathcal{K}_1), \mathcal{I}(\mathcal{K}_2), \ldots, \mathcal{I}(\mathcal{K}_n)) = \left(\oplus_{i=1}^{n}\zeta_i\underline{\mathcal{I}_\delta}(\mathcal{K}_i), \oplus_{i=1}^{n}\zeta_i\overline{\mathcal{I}_\delta}(\mathcal{K}_i)\right)$$

Based on above definition 9, the aggregated result for IFROWA operator is illustrated in Theorem 3.

Theorem 3: Let the collection $\mathfrak{I}(\mathcal{K}_i) = \left(\underline{\mathfrak{I}}(\mathcal{K}_i), \overline{\mathfrak{I}}(\mathcal{K}_i)\right)$ $(i = 1, 2, \ldots, n)$ of IFRVs with weight vectors $\zeta = (\zeta_1, \zeta_2, \ldots, \zeta_n)^T$. Then IFROWA operator is determined as:

$$\begin{aligned} &\text{IFROWA}(\mathfrak{I}(\mathcal{K}_1), \mathfrak{I}(\mathcal{K}_1), \ldots, \mathfrak{I}(\mathcal{K}_n)) \\ &= \left[\oplus_{i=1}^n \zeta_i \underline{\mathfrak{I}_\delta}(\mathcal{K}_i), \oplus_{i=1}^n \zeta_i \overline{\mathfrak{I}_\delta}(\mathcal{K}_i)\right] \\ &= \left[\begin{array}{c} \left(1-\prod_{i=1}^n\left(1-\underline{\mu_{\delta i}}\right)^{\zeta_i}, \prod_{i=1}^n\left(\underline{\gamma_{\delta i}}\right)^{\zeta_i}\right), \\ \left(1-\prod_{i=1}^n\left(1-\overline{\mu_{\delta i}}\right)^{\zeta_i}, \prod_{i=1}^n\left(\overline{\gamma_{\delta i}}\right)^{\zeta_i}\right) \end{array}\right], \end{aligned}$$

where $\mathfrak{I}_\delta(\mathcal{K}_i) = \left(\underline{\mathfrak{I}_\delta}(\mathcal{K}_i), \overline{\mathfrak{I}_\delta}(\mathcal{K}_i)\right)$ represents the largest value of permutation from the collection of IFRVs.

Proof: Follow from Theorem 1

Some important properties of IFROWA operator is illustrated in Theorem 4.

Theorem 4: Consider $\mathfrak{I}(\mathcal{K}_i) = \left(\underline{\mathfrak{I}}(\mathcal{K}_i), \overline{\mathfrak{I}}(\mathcal{K}_i)\right)$ $(i = 1, 2, \ldots, n)$ be the collection of IFRVs with weight vectors $\zeta = (\zeta_1, \zeta_2, \ldots, \zeta_n)^T$ with $\sum_{i=1}^n \zeta_i = 1$ and $0 \le \zeta_i \le 1$. Then some important properties of IFROWA operator are described as:

(i) **(Idempotency):** If $\mathfrak{I}(\mathcal{K}_i) = \mathcal{P}(\mathcal{K})$ for all $i = 1, 2, \ldots, n$ where $\mathcal{P}(\mathcal{K}) = \left(\underline{\mathcal{P}}(\mathcal{K}), \overline{\mathcal{P}}(\mathcal{K})\right) = \left((\underline{d}, \underline{e}), (\overline{d}, \overline{e})\right)$. Then $IFROWA(\mathfrak{I}(\mathcal{K}_1), \mathfrak{I}(\mathcal{K}_2), \ldots, \mathfrak{I}(\mathcal{K}_n)) = \mathcal{P}(\mathcal{K})$.

(ii) **(Boundedness):** Let $(\mathfrak{I}(\mathcal{K}))^- = \left(\min_i \underline{\mathfrak{I}}(\mathcal{K}_i), \max_i \overline{\mathfrak{I}}(\mathcal{K}_i)\right)$ and $(\mathfrak{I}(\mathcal{K}))^+ = \left(\max_i \underline{\mathfrak{I}}(\mathcal{K}_i), \min_j \overline{\mathfrak{I}}(\mathcal{K}_i)\right)$. Then $(\mathfrak{I}(\mathcal{K}))^- \le IFROWA(\mathfrak{I}(\mathcal{K}_1), \mathfrak{I}(\mathcal{K}_2), \ldots, \mathfrak{I}(\mathcal{K}_n)) \le (\mathfrak{I}(\mathcal{K}_i))^+$.

(iii) **(Monotonicity):** Let $\mathcal{P}(\mathcal{L}_i) = \left(\underline{\mathcal{P}}(\mathcal{L}_i), \overline{\mathcal{P}}(\mathcal{L}_i)\right)$ $(i = 1, 2, \ldots, n)$ be another collection of IFRVs such that $\underline{\mathcal{P}}(\mathcal{L}_i) \le \underline{\mathfrak{I}}(\mathcal{K}_i)$ and $\overline{\mathcal{P}}(\mathcal{L}_i) \le \overline{\mathfrak{I}}(\mathcal{K}_i)$. Then $IFROWA(\mathcal{P}(\mathcal{L}_1), \mathcal{P}(\mathcal{L}_2), \ldots, \mathcal{P}(\mathcal{L}_n)) \le IFROWA(\mathfrak{I}(\mathcal{K}_1), \mathfrak{I}(\mathcal{K}_2), \ldots, \mathfrak{I}(\mathcal{K}_n))$.

(iv) **(Shift invariance):** Consider another IFRV $\mathcal{P}(\mathcal{L}) = \left(\underline{\mathcal{P}}(\mathcal{L}), \overline{\mathcal{P}}(\mathcal{L})\right) = \left((\underline{d}, \underline{e}), (\overline{d}, \overline{e})\right)$ Then $IFROWA\left(\begin{array}{c}\mathfrak{I}(\mathcal{K}_1) \oplus \mathcal{P}(\mathcal{L}), \mathfrak{I}(\mathcal{K}_2) \oplus \mathcal{P}(\mathcal{L}), \ldots, \\ \mathfrak{I}(\mathcal{K}_n) \oplus \mathcal{P}(\mathcal{L})\end{array}\right) = IFROWA(\mathfrak{I}(\mathcal{K}_1), \mathfrak{I}(\mathcal{K}_2), \ldots, \mathfrak{I}(\mathcal{K}_n)) \oplus \mathcal{P}(\mathcal{L})$.

(v) **(Homogeneity):** For any real number $\lambda > 0$; $IFROWA(\lambda\mathfrak{I}(\mathcal{K}_1), \lambda\mathfrak{I}(\mathcal{K}_2), \ldots, \lambda\mathfrak{I}(\mathcal{K}_n)) = \lambda IFROWA(\mathfrak{I}(\mathcal{K}_1), \mathfrak{I}(\mathcal{K}_2), \ldots, \mathfrak{I}(\mathcal{K}_n))$

(vi) **(Commutativity):** Let $\mathfrak{I}'(\mathcal{K}_i) = \left(\underline{\mathfrak{I}'}(\mathcal{K}_i), \overline{\mathfrak{I}'}(\mathcal{K}_i)\right)$ $(i = 1, 2, \ldots, n)$ be any permutation of $\mathfrak{I}(\mathcal{K}_i) = \left(\underline{\mathfrak{I}}(\mathcal{K}_i), \overline{\mathfrak{I}}(\mathcal{K}_i)\right)$. Then $IFROWA(\mathfrak{I}(\mathcal{K}_1), \mathfrak{I}(\mathcal{K}_2), \ldots, \mathfrak{I}(\mathcal{K}_n)) = IFROWA\left(\mathfrak{I}'(\mathcal{K}_1), \mathfrak{I}'(\mathcal{K}_2), \ldots, \mathfrak{I}'(\mathcal{K}_n)\right)$.

Proof: Proof follows from Theorem 2.

C. INTUITIONISTIC FUZZY ROUGH HYBRID AVERAGING OPERATOR

In this subsection we will originate the notion of IFRHA operator, which weight both the value and their ordered position of an IF arguments at the same time. The important properties of the initiated operator presented in detail.

Definition 10: Consider the collection $\mathfrak{I}(\mathcal{K}_i) = \left(\underline{\mathfrak{I}}(\mathcal{K}_i), \overline{\mathfrak{I}}(\mathcal{K}_i)\right)$ $(i = 1, 2, \ldots, n)$ of IFRVs with weight vector $w = (w_1, w_2, \ldots, w_n)^T$ such that $\sum_{i=1}^n w_i = 1$ and $0 \le w_i \le 1$. Let $\zeta = (\zeta_1, \zeta_2, \ldots, \zeta_n)^T$ such that $\sum_{i=1}^n \zeta_i = 1$ and $0 \le \zeta_i \le 1$ be the associated weight vector of the given collection of IFRVs. Then IFRHA operator is determined as:

$$\begin{aligned} &\text{IFRHA}(\mathfrak{I}(\mathcal{K}_1), \mathfrak{I}(\mathcal{K}_2), \ldots, \mathfrak{I}(\mathcal{K}_n)) \\ &= \oplus_{i=1}^n \zeta_i \ddot{\mathfrak{I}}_\delta(\mathcal{K}_i) \\ &= \left[\oplus_{i=1}^n \zeta_i \underline{\ddot{\mathfrak{I}}_\delta}(\mathcal{K}_i), \oplus_{i=1}^n \zeta_i \overline{\ddot{\mathfrak{I}}_\delta}(\mathcal{K}_i)\right] \end{aligned}$$

Based on above definition 10, the aggregated result for IFRHA operator is illustrated in Theorem 5.

Theorem 5: Let the collection $\mathfrak{I}(\mathcal{K}_i) = \left(\underline{\mathfrak{I}}(\mathcal{K}_i), \overline{\mathfrak{I}}(\mathcal{K}_i)\right)$ $(i = 1, 2, \ldots, n)$ of IFRVs with weight vector $w = (w_1, w_2, \ldots, w_n)^T$ such that $\sum_{i=1}^n w_i = 1$ and $0 \le w_i \le 1$. Let $\zeta = (\zeta_1, \zeta_2, \ldots, \zeta_n)^T$ such that $\sum_{i=1}^n \zeta_i = 1$ and $0 \le \zeta_i \le 1$ be the associated weight vector of the given collection of IFRVs. Then IFRHA operator is determined as:

$$\begin{aligned} &\text{IFRHA}(\mathfrak{I}(\mathcal{K}_1), \mathfrak{I}(\mathcal{K}_1), \ldots, \mathfrak{I}(\mathcal{K}_n)) \\ &= \oplus_{i=1}^n \zeta_i \ddot{\mathfrak{I}}_\delta(\mathcal{K}_i) = \left[\oplus_{i=1}^n \zeta_i \underline{\ddot{\mathfrak{I}}_\delta}(\mathcal{K}_i), \oplus_{i=1}^n \zeta_i \overline{\ddot{\mathfrak{I}}_\delta}(\mathcal{K}_i)\right] \\ &= \left[\begin{array}{c} \left(1-\prod_{i=1}^n\left(1-\underline{\ddot{\mu}_{\delta i}}\right)^{\zeta_i}, \prod_{i=1}^n\left(\underline{\ddot{\gamma}_{\delta i}}\right)^{\zeta_i}\right), \\ \left(1-\prod_{i=1}^n\left(1-\overline{\ddot{\mu}_{\delta i}}\right)^{\zeta_i}, \prod_{i=1}^n\left(\overline{\ddot{\gamma}_{\delta i}}\right)^{\zeta_i}\right) \end{array}\right], \end{aligned}$$

where $\ddot{\mathfrak{I}}_\delta(\mathcal{K}_i) = nw_i\mathfrak{I}(\mathcal{K}_i) = \left(nw_i\underline{\mathfrak{I}}(\mathcal{K}_i), nw_i\overline{\mathfrak{I}}(\mathcal{K}_i)\right)$ represents the largest value of permutation from the collection of IFRVs and n denotes the balancing coefficient.

Proof: Follow from Theorem 1.

Especially, if $w = \left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)^T$, then the developed IFRHA operator reduced to IFROWA operator.

Some important properties of IFRHA operator is illustrated in Theorem 6.

Theorem 6: Consider the collection $\mathfrak{I}(\mathcal{K}_i) = \left(\underline{\mathfrak{I}}(\mathcal{K}_i), \overline{\mathfrak{I}}(\mathcal{K}_i)\right)$ $(i = 1, 2, \ldots, n)$ of IFRVs with weight vectors $w = (w_1, w_2, \ldots, w_n)^T$ such that $\sum_{i=1}^n w_i = 1$ and $0 \le w_i \le 1$. Let $\zeta = (\zeta_1, \zeta_2, \ldots, \zeta_n)^T$ such that $\sum_{i=1}^n \zeta_i = 1$ and $0 \le \zeta_i \le 1$ be the associated weight vector of the given collection of IFRVs. Then some important properties of IFRHA operator are described as:

(i) **(Idempotency):** If $\mathcal{I}(\mathcal{K}_i) = \mathcal{P}(\mathcal{K})$ for all $i = 1, 2, \ldots, n$ where $\mathcal{P}(\mathcal{K}) = \left(\underline{\mathcal{P}}(\mathcal{K}), \overline{\mathcal{P}}(\mathcal{K})\right) = \left((\underline{d}, \underline{e}), (\overline{d}, \overline{e})\right)$. Then
$IFRHA(\mathcal{I}(\mathcal{K}_1), \mathcal{I}(\mathcal{K}_2), \ldots, \mathcal{I}(\mathcal{K}_n)) = \mathcal{P}(\mathcal{K})$.

(ii) **(Boundedness):** Let $(\mathcal{I}(\mathcal{K}))^- = \left(\min_i \underline{\mathcal{I}}(\mathcal{K}_i), \max_i \overline{\mathcal{I}}(\mathcal{K}_i)\right)$ and $(\mathcal{I}(\mathcal{K}))^+ = \left(\max_i \underline{\mathcal{I}}(\mathcal{K}_i), \min_j \overline{\mathcal{I}}(\mathcal{K}_i)\right)$. Then
$(\mathcal{I}(\mathcal{K}))^- \leq IFRHA(\mathcal{I}(\mathcal{K}_1), \mathcal{I}(\mathcal{K}_2), \ldots, \mathcal{I}(\mathcal{K}_n)) \leq (\mathcal{I}(\mathcal{K}_i))^+$.

(iii) **(Monotonicity):** Let $\mathcal{P}(\mathcal{L}_i) = \left(\underline{\mathcal{P}}(\mathcal{L}_i), \overline{\mathcal{P}}(\mathcal{L}_i)\right)$ $(i = 1, 2, \ldots, n)$ be another collection of IFRVs such that $\underline{\mathcal{P}}(\mathcal{L}_i) \leq \underline{\mathcal{I}}(\mathcal{K}_i)$ and $\overline{\mathcal{P}}(\mathcal{L}_i) \leq \overline{\mathcal{I}}(\mathcal{K}_i)$. Then
$IFRHA(\mathcal{P}(\mathcal{L}_1), \mathcal{P}(\mathcal{L}_2), \ldots, \mathcal{P}(\mathcal{L}_n))$
$\leq IFRHA(\mathcal{I}(\mathcal{K}_1), \mathcal{I}(\mathcal{K}_2), \ldots, \mathcal{I}(\mathcal{K}_n))$.

(iv) **(Shift invariance):** Consider another IFRV $\mathcal{P}(\mathcal{L}) = \left(\underline{\mathcal{P}}(\mathcal{L}), \overline{\mathcal{P}}(\mathcal{L})\right) = \left((\underline{d}, \underline{e}), (\overline{d}, \overline{e})\right)$. Then
$$IFRHA\begin{pmatrix} \mathcal{I}(\mathcal{K}_1) \oplus \mathcal{P}(\mathcal{L}), \mathcal{I}(\mathcal{K}_2) \oplus \mathcal{P}(\mathcal{L}), \ldots, \\ \mathcal{I}(\mathcal{K}_n) \oplus \mathcal{P}(\mathcal{L}) \end{pmatrix}$$
$= IFRHA(\mathcal{I}(\mathcal{K}_1), \mathcal{I}(\mathcal{K}_2), \ldots, \mathcal{I}(\mathcal{K}_n)) \oplus \mathcal{P}(\mathcal{L})$.

(v) **(Homogeneity):** For any real number $\lambda > 0$; $IFRHA(\lambda\mathcal{I}(\mathcal{K}_1), \lambda\mathcal{I}(\mathcal{K}_2), \ldots, \lambda\mathcal{I}(\mathcal{K}_n)) = \lambda IFRHA(\mathcal{I}(\mathcal{K}_1), \mathcal{I}(\mathcal{K}_2), \ldots, \mathcal{I}(\mathcal{K}_n))$.

Proof: Proof follows from Theorem 2.

V. INTUITIONISTIC FUZZY ROUGH GEOMETRIC AGGREGATION OPERATOR

This section consists of the detailed study of IF rough geometric operator by embedding the concept of rough sets into IF geometric operator. Then some important properties of the investigated operators are presented in detail.

A. INTUITIONISTIC FUZZY ROUGH WEIGHTED GEOMETRIC OPERATOR

This subsection is devoted for the study of IFRWG aggregation operator and described its desirable characteristics.

Definition 11: Consider the collection $\mathcal{I}(\mathcal{K}_i) = \left(\underline{\mathcal{I}}(\mathcal{K}_i), \overline{\mathcal{I}}(\mathcal{K}_i)\right)$ $(i = 1, 2, \ldots, n)$ of IFRVs with weight vector $\zeta = (\zeta_1, \zeta_2, \ldots, \zeta_n)^T$ such that $\sum_{i=1}^{n} \zeta_i = 1$ and $0 \leq \zeta_i \leq 1$. The IFRWG operator is described as:

$$\mathrm{IFRWG}(\mathcal{I}(\mathcal{K}_1), \mathcal{I}(\mathcal{K}_2), \ldots, \mathcal{I}(\mathcal{K}_n)) = \left[\oplus_{i=1}^{n} \left(\underline{\mathcal{I}}(\mathcal{K}_i)\right)^{\zeta_i}, \oplus_{i=1}^{n} \left(\overline{\mathcal{I}}(\mathcal{K}_i)\right)^{\zeta_i}\right]$$

Based on analysis of Definition 11, the aggregated result for IFRWG operator is illustrated in Theorem 7.

Theorem 7: Let the collection $\mathcal{I}(\mathcal{K}_i) = \left(\underline{\mathcal{I}}(\mathcal{K}_i), \overline{\mathcal{I}}(\mathcal{K}_i)\right)$ $(i = 1, 2, \ldots, n)$ of IFRVs with weight vectors $\zeta = (\zeta_1, \zeta_2, \ldots, \zeta_n)^T$. Then IFRWG operator is determined as:

$$\mathrm{IFRWG}(\mathcal{I}(\mathcal{K}_1), \mathcal{I}(\mathcal{K}_1), \ldots, \mathcal{I}(\mathcal{K}_n)) = \left[\oplus_{i=1}^{n} \left(\underline{\mathcal{I}}(\mathcal{K}_i)\right)^{\zeta_i}, \oplus_{i=1}^{n} \left(\overline{\mathcal{I}}(\mathcal{K}_i)\right)^{\zeta_i}\right]$$

$$= \left[\begin{array}{c} \left(\prod_{i=1}^{n} \left(\underline{\mu_i}\right)^{\zeta_i}, 1 - \prod_{i=1}^{n} \left(1 - \underline{\gamma_i}\right)^{\zeta_i}\right), \\ \left(\prod_{i=1}^{n} (\overline{\mu_i})^{\zeta_i}, 1 - \prod_{i=1}^{n} (1 - \overline{\gamma_i})^{\zeta_i}\right) \end{array}\right].$$

Proof: Proof is omitted here and follow from Theorem 1.

From the above analysis $\underline{\mathcal{I}}(\mathcal{K})$ and $\overline{\mathcal{I}}(\mathcal{K})$ are IFRVs. So, by Definition 11, $\oplus_{i=1}^{n} \left(\underline{\mathcal{I}}(\mathcal{K}_i)\right)^{\zeta_i}$ and $\oplus_{i=1}^{n} \left(\overline{\mathcal{I}}(\mathcal{K}_i)\right)^{\zeta_i}$ are also IFRVs. Therefore IFRWG $(\mathcal{I}(\mathcal{K}_1), \ldots, \mathcal{I}(\mathcal{K}_n))$ is also an IFRV under IF approximation space $(\mathcal{N}, \mathcal{I})$.

Some important properties of IFRWG operator are described in Theorem 8.

Theorem 8: Consider the collection $\mathcal{I}(\mathcal{K}_i) = \left(\underline{\mathcal{I}}(\mathcal{K}_i), \overline{\mathcal{I}}(\mathcal{K}_i)\right)$ $(i = 1, 2, \ldots, n)$ of IFRVs with weight vectors $\zeta = (\zeta_1, \zeta_2, \ldots, \zeta_n)^T$ with $\sum_{i=1}^{n} \zeta_i = 1$ and $0 \leq \zeta_i \leq 1$. Then some important properties of IFRWG operator are described as:

(i) **(Idempotency):** If $\mathcal{I}(\mathcal{K}_i) = \mathcal{P}(\mathcal{K})$ for all $i = 1, 2, \ldots, n$ where $\mathcal{P}(\mathcal{K}) = \left(\underline{\mathcal{P}}(\mathcal{K}), \overline{\mathcal{P}}(\mathcal{K})\right) = \left((\underline{d}, \underline{e}), (\overline{d}, \overline{e})\right)$. Then
$IFRWG(\mathcal{I}(\mathcal{K}_1), \mathcal{I}(\mathcal{K}_2), \ldots, \mathcal{I}(\mathcal{K}_n)) = \mathcal{P}(\mathcal{K})$.

(ii) **(Boundedness):** Let $(\mathcal{I}(\mathcal{K}))^- = \left(\min_i \underline{\mathcal{I}}(\mathcal{K}_i), \max_i \overline{\mathcal{I}}(\mathcal{K}_i)\right)$ and $(\mathcal{I}(\mathcal{K}))^+ = \left(\max_i \underline{\mathcal{I}}(\mathcal{K}_i), \min_j \overline{\mathcal{I}}(\mathcal{K}_i)\right)$. Then
$(\mathcal{I}(\mathcal{K}))^- \leq IFRWG(\mathcal{I}(\mathcal{K}_1), \mathcal{I}(\mathcal{K}_2), \ldots, \mathcal{I}(\mathcal{K}_n)) \leq (\mathcal{I}(\mathcal{K}_i))^+$.

(iii) **(Monotonicity):** Let $\mathcal{P}(\mathcal{L}_i) = \left(\underline{\mathcal{P}}(\mathcal{L}_i), \overline{\mathcal{P}}(\mathcal{L}_i)\right)$ $(i = 1, 2, \ldots, n)$ be another collection of IFRVs such that $\underline{\mathcal{P}}(\mathcal{L}_i) \leq \underline{\mathcal{I}}(\mathcal{K}_i)$ and $\overline{\mathcal{P}}(\mathcal{L}_i) \leq \overline{\mathcal{I}}(\mathcal{K}_i)$. Then
$IFRWG(\mathcal{P}(\mathcal{L}_1), \mathcal{P}(\mathcal{L}_2), \ldots, \mathcal{P}(\mathcal{L}_n))$
$\leq IFRWG(\mathcal{I}(\mathcal{K}_1), \mathcal{I}(\mathcal{K}_2), \ldots, \mathcal{I}(\mathcal{K}_n))$.

(iv) **(Shift invariance):** Consider another IFRV $\mathcal{P}(\mathcal{L}) = \left(\underline{\mathcal{P}}(\mathcal{L}), \overline{\mathcal{P}}(\mathcal{L})\right) = \left((\underline{d}, \underline{e}), (\overline{d}, \overline{e})\right)$. Then
$$IFRWG\begin{pmatrix} \mathcal{I}(\mathcal{K}_1) \oplus \mathcal{P}(\mathcal{L}), \mathcal{I}(\mathcal{K}_2) \oplus \mathcal{P}(\mathcal{L}), \ldots, \\ \mathcal{I}(\mathcal{K}_n) \oplus \mathcal{P}(\mathcal{L}) \end{pmatrix} =$$
$IFRWG(\mathcal{I}(\mathcal{K}_1), \mathcal{I}(\mathcal{K}_2), \ldots, \mathcal{I}(\mathcal{K}_n)) \oplus \mathcal{P}(\mathcal{L})$.

(v) **(Homogeneity):** For any real number $\lambda > 0$; $IFRWG(\lambda\mathcal{I}(\mathcal{K}_1), \lambda\mathcal{I}(\mathcal{K}_2), \ldots, \lambda\mathcal{I}(\mathcal{K}_n)) = \lambda IFRWG(\mathcal{I}(\mathcal{K}_1), \mathcal{I}(\mathcal{K}_2), \ldots, \mathcal{I}(\mathcal{K}_n))$.

(vi) **(Commutativity):** Let $\mathcal{I}'(\mathcal{K}_i) = \left(\underline{\mathcal{I}'}(\mathcal{K}_i), \overline{\mathcal{I}'}(\mathcal{K}_i)\right)$ $(i = 1, 2, \ldots, n)$ be any permutation of $\mathcal{I}(\mathcal{K}_i) = \left(\underline{\mathcal{I}}(\mathcal{K}_i), \overline{\mathcal{I}}(\mathcal{K}_i)\right)$. Then
$IFRWG(\mathcal{I}(\mathcal{K}_1), \mathcal{I}(\mathcal{K}_2), \ldots, \mathcal{I}(\mathcal{K}_n)) = IFRWG\left(\mathcal{I}'(\mathcal{K}_1), \mathcal{I}'(\mathcal{K}_2), \ldots, \mathcal{I}'(\mathcal{K}_n)\right)$.

Proof: Proof are easy and follows from Theorem 2.

B. INTUITIONISTIC FUZZY ROUGH ORDERED WEIGHTED GEOMETRIC OPERATOR

Here we shall investigate the concept of IFROWG aggregation operators and discussed its important properties.

Definition 12: Consider the collection $\mathcal{I}(\mathcal{K}_i) = \left(\underline{\mathcal{I}}(\mathcal{K}_i), \overline{\mathcal{I}}(\mathcal{K}_i)\right)$ $(i = 1, 2, \ldots, n)$ of IFRVs with weight vector $\zeta = (\zeta_1, \zeta_2, \ldots, \zeta_n)^T$ such that $\sum_{i=1}^{n} \zeta_i = 1$ and $0 \leq \zeta_i \leq 1$. The IFROWG operator is determined as:

$$\text{IFROWG}(\mathcal{I}(\mathcal{K}_1), \mathcal{I}(\mathcal{K}_2), \ldots, \mathcal{I}(\mathcal{K}_n)) = \left[\oplus_{i=1}^{n} \left(\underline{\mathcal{I}_\delta}(\mathcal{K}_i)\right)^{\zeta_i}, \oplus_{i=1}^{n} \left(\overline{\mathcal{I}_\delta}(\mathcal{K}_i)\right)^{\zeta_i}\right]$$

Based on above Definition 12, the aggregated result for IFROWG operator is illustrated in Theorem 9.

Theorem 9: Let the collection $\mathcal{I}(\mathcal{K}_i) = \left(\underline{\mathcal{I}}(\mathcal{K}_i), \overline{\mathcal{I}}(\mathcal{K}_i)\right)$ $(i = 1, 2, \ldots, n)$ of IFRVs with weight vectors $\zeta = (\zeta_1, \zeta_2, \ldots, \zeta_n)^T$. Then IFROWG operator is determined as:

$$\begin{aligned} &\text{IFROWA}(\mathcal{I}(\mathcal{K}_1), \mathcal{I}(\mathcal{K}_1), \ldots, \mathcal{I}(\mathcal{K}_n)) \\ &= \left[\oplus_{i=1}^{n} \left(\underline{\mathcal{I}_\delta}(\mathcal{K}_i)\right)^{\zeta_i}, \oplus_{i=1}^{n} \left(\overline{\mathcal{I}_\delta}(\mathcal{K}_i)\right)^{\zeta_i}\right] \\ &= \left[\begin{array}{c} \left(\prod_{i=1}^{n} \left(\underline{\mu_{\delta i}}\right)^{\zeta_i}, 1 - \prod_{i=1}^{n} \left(1 - \underline{\gamma_{\delta i}}\right)^{\zeta_i}\right), \\ \left(\prod_{i=1}^{n} \left(\overline{\mu_{\delta i}}\right)^{\zeta_i}, 1 - \prod_{i=1}^{n} \left(1 - \overline{\gamma_{\delta i}}\right)^{\zeta_i}\right) \end{array}\right], \end{aligned}$$

where $\mathcal{I}_\delta(\mathcal{K}_i) = \left(\underline{\mathcal{I}_\delta}(\mathcal{K}_i), \overline{\mathcal{I}_\delta}(\mathcal{K}_i)\right)$ represents the largest value of permutation from the collection of IFRVs.

Proof: Proof can be follow from Theorem 1.

Some important properties of IFROWG operator is developed in Theorem 10.

Theorem 10: Consider the collection $\mathcal{I}(\mathcal{K}_i) = \left(\underline{\mathcal{I}}(\mathcal{K}_i), \overline{\mathcal{I}}(\mathcal{K}_i)\right)$ $(i = 1, 2, \ldots, n)$ of IFRVs with weight vectors $\zeta = (\zeta_1, \zeta_2, \ldots, \zeta_n)^T$ with $\sum_{i=1}^{n} \zeta_i = 1$ and $0 \leq \zeta_i \leq 1$. Then some important properties of IFROWG operator are described as:

(i) **(Idempotency):** If $\mathcal{I}(\mathcal{K}_i) = \mathcal{P}(\mathcal{K})$ for all $i = 1, 2, \ldots, n$ where $\mathcal{P}(\mathcal{K}) = \left(\underline{\mathcal{P}}(\mathcal{K}), \overline{\mathcal{P}}(\mathcal{K})\right) = \left((\underline{d}, \underline{e}), (\overline{d}, \overline{e})\right)$. Then $IFROWG(\mathcal{I}(\mathcal{K}_1), \mathcal{I}(\mathcal{K}_2), \ldots, \mathcal{I}(\mathcal{K}_n)) = \mathcal{P}(\mathcal{K})$.

(ii) **(Boundedness):** Let $(\mathcal{I}(\mathcal{K}))^- = \left(\min_i \underline{\mathcal{I}}(\mathcal{K}_i), \max_i \overline{\mathcal{I}}(\mathcal{K}_i)\right)$ and $(\mathcal{I}(\mathcal{K}))^+ = \left(\max_i \underline{\mathcal{I}}(\mathcal{K}_i), \min_j \overline{\mathcal{I}}(\mathcal{K}_i)\right)$. Then $(\mathcal{I}(\mathcal{K}))^- \leq IFROWG(\mathcal{I}(\mathcal{K}_1), \mathcal{I}(\mathcal{K}_2), \ldots, \mathcal{I}(\mathcal{K}_n)) \leq (\mathcal{I}(\mathcal{K}_i))^+$.

(iii) **(Monotonicity):** Let $\mathcal{P}(\mathcal{L}_i) = \left(\underline{\mathcal{P}}(\mathcal{L}_i), \overline{\mathcal{P}}(\mathcal{L}_i)\right)$ $(i = 1, 2, \ldots, n)$ be another collection of IFRVs such that $\underline{\mathcal{P}}(\mathcal{L}_i) \leq \underline{\mathcal{I}}(\mathcal{K}_i)$ and $\overline{\mathcal{P}}(\mathcal{L}_i) \leq \overline{\mathcal{I}}(\mathcal{K}_i)$. Then $IFROWG(\mathcal{P}(\mathcal{L}_1), \mathcal{P}(\mathcal{L}_2), \ldots, \mathcal{P}(\mathcal{L}_n)) \leq IFROWG(\mathcal{I}(\mathcal{K}_1), \mathcal{I}(\mathcal{K}_2), \ldots, \mathcal{I}(\mathcal{K}_n))$.

(iv) **(Shift invariance):** Consider another IFRV $\mathcal{P}(\mathcal{L}) = \left(\underline{\mathcal{P}}(\mathcal{L}), \overline{\mathcal{P}}(\mathcal{L})\right) = \left((\underline{d}, \underline{e}), (\overline{d}, \overline{e})\right)$. Then $IFROWG\begin{pmatrix} \mathcal{I}(\mathcal{K}_1) \oplus \mathcal{P}(\mathcal{L}), \mathcal{I}(\mathcal{K}_2) \oplus \mathcal{P}(\mathcal{L}), \ldots, \\ \mathcal{I}(\mathcal{K}_n) \oplus \mathcal{P}(\mathcal{L}) \end{pmatrix} = IFROWG(\mathcal{I}(\mathcal{K}_1), \mathcal{I}(\mathcal{K}_2), \ldots, \mathcal{I}(\mathcal{K}_n)) \oplus \mathcal{P}(\mathcal{L})$.

(v) **(Homogeneity):** For any real number $\lambda > 0$; $IFROWG(\lambda\mathcal{I}(\mathcal{K}_1), \lambda\mathcal{I}(\mathcal{K}_2), \ldots, \lambda\mathcal{I}(\mathcal{K}_n)) = \lambda IFROWG(\mathcal{I}(\mathcal{K}_1), \mathcal{I}(\mathcal{K}_2), \ldots, \mathcal{I}(\mathcal{K}_n))$

(vi) **(Commutativity):** Let $\mathcal{I}'(\mathcal{K}_i) = \left(\underline{\mathcal{I}'}(\mathcal{K}_i), \overline{\mathcal{I}'}(\mathcal{K}_i)\right)$ $(i = 1, 2, \ldots, n)$ be any permutation of $\mathcal{I}(\mathcal{K}_i) = \left(\underline{\mathcal{I}}(\mathcal{K}_i), \overline{\mathcal{I}}(\mathcal{K}_i)\right)$. Then $IFROWG(\mathcal{I}(\mathcal{K}_1), \mathcal{I}(\mathcal{K}_2), \ldots, \mathcal{I}(\mathcal{K}_n)) = IFROWG\left(\mathcal{I}'(\mathcal{K}_1), \mathcal{I}'(\mathcal{K}_2), \ldots, \mathcal{I}'(\mathcal{K}_n)\right)$.

Proof: Proof are easy and follows from Theorem 2.

C. INTUITIONISTIC FUZZY ROUGH HYBRID GEOMETRIC OPERATOR

In this section we will present the notion of IFRHG operator, which weight both the value and their ordered position of an IF arguments at the same time. The important properties of the developed operator presented in detail.

Definition 13: Consider the collection $\mathcal{I}(\mathcal{K}_i) = \left(\underline{\mathcal{I}}(\mathcal{K}_i), \overline{\mathcal{I}}(\mathcal{K}_i)\right)$ $(i = 1, 2, \ldots, n)$ of IFRVs with weight vector $w = (w_1, w_2, \ldots, w_n)^T$ such that $\sum_{i=1}^{n} w_i = 1$ and $0 \leq w_i \leq 1$. Let $\zeta = (\zeta_1, \zeta_2, \ldots, \zeta_n)^T$ such that $\sum_{i=1}^{n} \zeta_i = 1$ and $0 \leq \zeta_i \leq 1$ be the associated weight vector of the given collection of IFRVs. Then IFRHG operator is determined as:

$$\begin{aligned} &\text{IFRHG}(\mathcal{I}(\mathcal{K}_1), \mathcal{I}(\mathcal{K}_2), \ldots, \mathcal{I}(\mathcal{K}_n)) \\ &= \oplus_{i=1}^{n} \left(\ddot{\mathcal{I}}_\delta(\mathcal{K}_i)\right)^{\zeta_i} \\ &= \left[\oplus_{i=1}^{n} \left(\underline{\ddot{\mathcal{I}}_\delta}(\mathcal{K}_i)\right)^{\zeta_i}, \oplus_{i=1}^{n} \left(\overline{\ddot{\mathcal{I}}_\delta}(\mathcal{K}_i)\right)^{\zeta_i}\right] \end{aligned}$$

Based on above definition 13, the aggregated result for IFRHG operator is illustrated in Theorem 11.

Theorem 11: Let the collection $\mathcal{I}(\mathcal{K}_i) = \left(\underline{\mathcal{I}}(\mathcal{K}_i), \overline{\mathcal{I}}(\mathcal{K}_i)\right)$ $(i = 1, 2, \ldots, n)$ of IFRVs with weight vector $w = (w_1, w_2, \ldots, w_n)^T$ such that $\sum_{i=1}^{n} w_i = 1$ and $0 \leq w_i \leq 1$. Let $\zeta = (\zeta_1, \zeta_2, \ldots, \zeta_n)^T$ such that $\sum_{i=1}^{n} \zeta_i = 1$ and $0 \leq \zeta_i \leq 1$ be the associated weight vector of the given collection of IFRVs. Then IFRHA operator is determined as:

$$\begin{aligned} &\text{IFRHG}(\mathcal{I}(\mathcal{K}_1), \mathcal{I}(\mathcal{K}_1), \ldots, \mathcal{I}(\mathcal{K}_n)) = \oplus_{i=1}^{n} \left(\ddot{\mathcal{I}}_\delta(\mathcal{K}_i)\right)^{\zeta_i} \\ &= \left[\oplus_{i=1}^{n} \left(\underline{\ddot{\mathcal{I}}_\delta}(\mathcal{K}_i)\right)^{\zeta_i}, \oplus_{i=1}^{n} \left(\overline{\ddot{\mathcal{I}}_\delta}(\mathcal{K}_i)\right)^{\zeta_i}\right] \\ &= \left[\begin{array}{c} \left(\prod_{i=1}^{n} \left(\underline{\ddot{\mu}_{\delta i}}\right)^{\zeta_i}, 1 - \prod_{i=1}^{n} \left(1 - \underline{\ddot{\gamma}_{\delta i}}\right)^{\zeta_i}\right), \\ \left(\prod_{i=1}^{n} \left(\overline{\ddot{\mu}_{\delta i}}\right)^{\zeta_i}, 1 - \prod_{i=1}^{n} \left(1 - \overline{\ddot{\gamma}_{\delta i}}\right)^{\zeta_i}\right) \end{array}\right], \end{aligned}$$

where $\ddot{\mathcal{I}}_\delta(\mathcal{K}_i) = (\mathcal{I}(\mathcal{K}_i))^{nw_i} = \left(\left(\underline{\mathcal{I}}(\mathcal{K}_i)\right)^{nw_i}, \left(\overline{\mathcal{I}}(\mathcal{K}_i)\right)^{nw_i}\right)$ represents the largest value of permutation from the collection of IFRVs and n denotes the balancing coefficient.

Proof: Follow from Theorem 1.

Especially, if $w = \left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)^T$, then the developed IFRHG operator reduced to IFROWG operator.

TABLE 2. IF rough evaluation information by $\mathcal{E}_1$.

	c_1	c_2	c_3	c_4	c_5
$\wp_1$	$((0.6,0.3),(0.8,0.2))$	$((0.8,0.1),(0.7,0.2))$	$((0.9,0.1),(0.4,0.3))$	$((0.9,0.1),(0.7,0.2))$	$((0.5,0.1),(0.4,0.2))$
$\wp_2$	$((0.5,0.2),(0.9,0.1))$	$((0.6,0.2),(0.4,0.3))$	$((0.4,0.1),(0.8,0.1))$	$((0.7,0.2),(0.5,0.1))$	$((0.4,0.1),(0.6,0.4))$
$\wp_3$	$((0.6,0.4),(0.5,0.3))$	$((0.6,0.3),(0.5,0.2))$	$((0.6,0.1),(0.4,0.3))$	$((0.4,0.5),(0.3,0.6))$	$((0.6,0.4),(0.4,0.3))$
$\wp_4$	$((0.7,0.1),(0.4,0.2))$	$((0.4,0.6),(0.5,0.4))$	$((0.2,0.1),(0.5,0.2))$	$((0.5,0.4),(0.3,0.5))$	$((0.5,0.3),(0.4,0.2))$

Some important properties of IFRHG operator is described in Theorem 12.

Theorem 12: Consider the collection $\mathfrak{J}(\mathcal{K}_i) = \left(\underline{\mathfrak{J}}(\mathcal{K}_i), \overline{\mathfrak{J}}(\mathcal{K}_i)\right)$ $(i = 1, 2, \ldots, n)$ of IFRVs with weight vectors $w = (w_1, w_2, \ldots, w_n)^T$ such that $\sum_{i=1}^{n} w_i = 1$ and $0 \leq w_i \leq 1$. Let $\zeta = (\zeta_1, \zeta_2, \ldots, \zeta_n)^T$ such that $\sum_{i=1}^{n} \zeta_i = 1$ and $0 \leq \zeta_i \leq 1$ be the associated weight vector of the given collection of IFRVs. Then some important properties of IFRHG operator are described as:

(i) **(Idempotency):** If $\mathfrak{J}(\mathcal{K}_i) = \mathcal{P}(\mathcal{K})$ for all $i = 1, 2, \ldots, n$ where $\mathcal{P}(\mathcal{K}) = \left(\underline{\mathcal{P}}(\mathcal{K}), \overline{\mathcal{P}}(\mathcal{K})\right) = \left((\underline{d}, \underline{e}), (\overline{d}, \overline{e})\right)$. Then $IFRHG(\mathfrak{J}(\mathcal{K}_1), \mathfrak{J}(\mathcal{K}_2), \ldots, \mathfrak{J}(\mathcal{K}_n)) = \mathcal{P}(\mathcal{K})$.

(ii) **(Boundedness):** Let $(\mathfrak{J}(\mathcal{K}))^- = \left(\min_i \underline{\mathfrak{J}}(\mathcal{K}_i), \max_i \overline{\mathfrak{J}}(\mathcal{K}_i)\right)$ and $(\mathfrak{J}(\mathcal{K}))^+ = \left(\max_i \underline{\mathfrak{J}}(\mathcal{K}_i), \min_j \overline{\mathfrak{J}}(\mathcal{K}_i)\right)$. Then $(\mathfrak{J}(\mathcal{K}))^- \leq IFRHG(\mathfrak{J}(\mathcal{K}_1), \mathfrak{J}(\mathcal{K}_2), \ldots, \mathfrak{J}(\mathcal{K}_n)) \leq (\mathfrak{J}(\mathcal{K}_i))^+$.

(iii) **(Monotonicity):** Let $\mathcal{P}(\mathcal{L}_i) = \left(\underline{\mathcal{P}}(\mathcal{L}_i), \overline{\mathcal{P}}(\mathcal{L}_i)\right)$ $(i = 1, 2, \ldots, n)$ be another collection of IFRVs such that $\underline{\mathcal{P}}(\mathcal{L}_i) \leq \underline{\mathfrak{J}}(\mathcal{K}_i)$ and $\overline{\mathcal{P}}(\mathcal{L}_i) \leq \overline{\mathfrak{J}}(\mathcal{K}_i)$. Then $IFRHG(\mathcal{P}(\mathcal{L}_1), \mathcal{P}(\mathcal{L}_2), \ldots, \mathcal{P}(\mathcal{L}_n)) \leq IFRHG(\mathfrak{J}(\mathcal{K}_1), \mathfrak{J}(\mathcal{K}_2), \ldots, \mathfrak{J}(\mathcal{K}_n))$.

(iv) **(Shift invariance):** Consider another IFRV $\mathcal{P}(\mathcal{L}) = \left(\underline{\mathcal{P}}(\mathcal{L}), \overline{\mathcal{P}}(\mathcal{L})\right) = \left((\underline{d}, \underline{e}), (\overline{d}, \overline{e})\right)$. Then $IFRHG\left(\begin{array}{c}\mathfrak{J}(\mathcal{K}_1) \oplus \mathcal{P}(\mathcal{L}), \mathfrak{J}(\mathcal{K}_2) \oplus \mathcal{P}(\mathcal{L}), \ldots, \\ \mathfrak{J}(\mathcal{K}_n) \oplus \mathcal{P}(\mathcal{L})\end{array}\right) = IFRHG(\mathfrak{J}(\mathcal{K}_1), \mathfrak{J}(\mathcal{K}_2), \ldots, \mathfrak{J}(\mathcal{K}_n)) \oplus \mathcal{P}(\mathcal{L})$.

(v) **(Homogeneity):** For any real number $\lambda > 0$; $IFRHG(\lambda\mathfrak{J}(\mathcal{K}_1), \lambda\mathfrak{J}(\mathcal{K}_2), \ldots, \lambda\mathfrak{J}(\mathcal{K}_n)) = \lambda IFRHG(\mathfrak{J}(\mathcal{K}_1), \mathfrak{J}(\mathcal{K}_2), \ldots, \mathfrak{J}(\mathcal{K}_n))$.

Proof: Proof follows Theorem 2.

VI. EDAS METHOD FOR MCGDM BASED ON ROUGH AGGREGATION OPERATORS BY USING IF INFORM-ATION

In this competitive environment, the complexity in DM problems grows with the intricacy of the socio-economic environment. So, in this scenario, it becomes more complicated for an expert to take an accurate and intelligent decision. In real life, it is intensively needed to fuse a group of professional experts' opinion to achieve more satisfactory and useful results by utilizing group decision making models. Therefore, MCGDM has the high potential and discipline process to improve and evaluate multiple conflicting criteria in all areas of decision making to get more satisfactory and feasible decision making result. Here, we will use EDAS method to solve MCGDM approach. The EDAS method was presented by Ghorabaee et al. [40]. It was based on *PDAS* and *NDAS* from *AvS*. Superior value of *PDAS* and inferior value of *NDAS* is considered the optimal choice. To study the hybrid structure of EDAS method with IFRVs, we get intuitionistic fuzzy rough EDAS (IFR-EDAS) method in which the experts provided their assessment values in the form of IFRVs. The basic steps of construction by utilizing the proposed approach under IF rough information are as follows.

Suppose a set of m alternatives is represented by $\mathcal{N} = \{\wp_1, \wp_2, \ldots, \wp_m\}$ and a set of n decision attributes is denoted by $\mathcal{C} = \{c_1, c_2, \ldots, c_n\}$. Let $\mathcal{D} = \{\mathcal{D}_1, \mathcal{D}_2, \ldots, \mathcal{D}_t\}$ be a set of t professional decision makers who presented their evaluation report for each alternative $\wp_i (i = 1, 2, \ldots, m)$ against their attributes $c_j (j = 1, 2, \ldots, n)$. Let $\zeta = (\zeta_1, \zeta_2, \ldots, \zeta_n)^T$ be the weight vector for attributes c_j and $\vartheta = (\vartheta_1, \vartheta_2, \ldots, \vartheta_t)^T$ be the weight vector for decision maker $\mathcal{D}_l (l = 1, 2, \ldots, t)$ such that $\sum_{j=1}^{n} \zeta_j = 1$, $\sum_{l=1}^{t} \vartheta_l = 1$ and $0 \leq \zeta_j, \vartheta_l \leq 1$. The classical algorithm for EDAS method with IF rough environment is described as:

Step 1: Collect the evaluation information of professional decision makers for each alternative $\wp_i$ against their attribute c_j and construct a decision matrix which is given as:

$$\mathbb{M} = \left[\mathfrak{J}\left(\mathcal{K}_{ij}^{l}\right)\right]_{m \times n}$$

where $\mathfrak{J}\left(\mathcal{K}_{ij}^{l}\right)$ represent the IFRVs of alternative $\wp_i$ against attributes c_j by the professional decision maker $\mathcal{D}_l$

Step 2: The collective information decision makers against their weight vector are aggregated by utilizing the proposed approach to get the aggregated decision matrix.

$$\mathbb{M} = \left[\mathfrak{J}(\mathcal{K}_{ij})\right]_{(m \times n)},$$

TABLE 3. IF rough evaluation information by $\mathcal{E}_2$.

	c_1	c_2	c_3	c_4	c_5
$\wp_1$	$((0.6,0.1),(0.3,0.2))$	$((0.8,0.2),(0.6,0.3))$	$((0.6,0.3),(0.7,0.1))$	$((0.5,0.1),(0.7,0.3))$	$((0.9,0.1),(0.8,0.1))$
$\wp_2$	$((0.4,0.1),(0.5,0.3))$	$((0.9,0.1),(0.4,0.2))$	$((0.4,0.5),(0.4,0.1))$	$((0.4,0.3),(0.2,0.7))$	$((0.8,0.2),(0.4,0.5))$
$\wp_3$	$((0.7,0.2),(0.2,0.7))$	$((0.4,0.2),(0.2,0.6))$	$((0.5,0.3),(0.5,0.3))$	$((0.2,0.7),(0.7,0.2))$	$((0.5,0.3),(0.3,0.1))$
$\wp_4$	$((0.5,0.5),(0.6,0.2))$	$((0.6,0.3),(0.4,0.3))$	$((0.3,0.1),(0.5,0.4))$	$((0.6,0.2),(0.4,0.5))$	$((0.6,0.3),(0.2,0.3))$

TABLE 4. IF rough evaluation information by $\mathcal{E}_3$.

	c_1	c_2	c_3	c_4	c_5
$\wp_1$	$((0.7,0.1),(0.2,0.1))$	$((0.8,0.2),(0.6,0.3))$	$((0.7,0.3),(0.6,0.2))$	$((0.5,0.2),(0.4,0.1))$	$((0.8,0.1),(0.7,0.2))$
$\wp_2$	$((0.7,0.2),(0.3,0.4))$	$((0.5,0.3),(0.4,0.3))$	$((0.5,0.1),(0.3,0.6))$	$((0.7,0.3),(0.5,0.1))$	$((0.6,0.3),(0.4,0.1))$
$\wp_3$	$((0.4,0.2),(0.5,0.1))$	$((0.9,0.1),(0.9,0.1))$	$((0.4,0.3),(0.2,0.6))$	$((0.8,0.2),(0.3,0.4))$	$((0.6,0.4),(0.3,0.2))$
$\wp_4$	$((0.5,0.1),(0.7,0.3))$	$((0.5,0.2),(0.4,0.6))$	$((0.3,0.2),(0.1,0.3))$	$((0.3,0.1),(0.4,0.2))$	$((0.4,0.2),(0.5,0.3))$

Step 3: Normalized the aggregated matrix $\mathbb{M} = [\mathcal{I}(\mathcal{K}_{ij})]_{(m\times n)}$ to $\mathbb{M}^n = \left[\mathcal{I}\left(\mathcal{K}_{ij}^n\right)\right]_{(m\times n)}$, that is

$$\mathbb{M}^n = \begin{cases} \mathcal{I}(\mathcal{K}_{ij}) = \left(\left(\underline{\mu_{ij}}, \underline{\gamma_{ij}}\right), \left(\overline{\mu_{ij}}, \overline{\gamma_{ij}}\right)\right) \\ \quad \textit{for benefit} \\ \mathcal{I}\left(\mathcal{K}_{ij}^n\right) = \mathcal{I}(\mathcal{K}_{ij})^c = \left(\left(\underline{\gamma_i}, \underline{\mu_i}\right), \left(\overline{\gamma_i}, \overline{\mu_i}\right)\right) \\ \quad \textit{for cost} \end{cases}$$

Step 4: Calculate the value of $Av\mathcal{S}$ by applying developed approached for all alternative under each attribute.

$$Av\mathcal{S} = [Av\mathcal{S}_j]_{1\times n} = \left[\frac{1}{m}\sum\nolimits_{i=1}^{m} \mathcal{I}\left(\mathcal{K}_{ij}^n\right)\right]_{1\times n},$$

This implies

$$\begin{aligned} Av\mathcal{S} &= [Av\mathcal{S}_j]_{1\times n} = \left[\frac{1}{m}\sum\nolimits_{i=1}^{m} \mathcal{I}\left(\mathcal{K}_{ij}^n\right)\right]_{1\times n} \\ &= \left[\begin{array}{l} \left(1-\prod_{i=1}^{m}\left(1-\underline{\mu_{ij}^n}\right)^{\frac{1}{m}}, \prod_{i=1}^{m}\left(\underline{\gamma_{ij}^n}\right)^{\frac{1}{m}}\right), \\ \wp\left(1-\prod_{i=1}^{m}\left(1-\overline{\mu_{ij}^n}\right)^{\frac{1}{m}}, \prod_{i=1}^{m}\left(\overline{\gamma_{ij}^n}\right)^{\frac{1}{m}}\right) \end{array}\right]_{1\times n} \end{aligned}$$

Step 5: Based on determined $Av\mathcal{S}$, we can calculate $PDA\mathcal{S}$ and $NDA\mathcal{S}$ by utilizing the below formula:

$$\begin{aligned} PDA\mathcal{S}_{ij} &= [PDA\mathcal{S}_{ij}]_{(m\times n)} \\ &= \frac{max\left(0, \left[\mathcal{S}\left(\mathcal{I}\left(\mathcal{K}_{ij}^n\right)\right) - \mathcal{S}(Av\mathcal{S}_j)\right]\right)}{\mathcal{S}(Av\mathcal{S}_j)}, \end{aligned}$$

$$\begin{aligned} NDA\mathcal{S} &= [NDA\mathcal{S}_{ij}]_{(m\times n)} \\ &= \frac{max\left(0, \left[\mathcal{S}(Av\mathcal{S}_j) - \mathcal{S}\left(\mathcal{I}\left(\mathcal{K}_{ij}^n\right)\right)\right]\right)}{\mathcal{S}(Av\mathcal{S}_j)}. \end{aligned}$$

Step 6: Next to calculate the positive weight distance ($\mathcal{S}P_i$) and negative weight distance ($\mathcal{S}N_i$)

$$\mathcal{S}P_i = \sum\nolimits_{j=1}^{n} \zeta_j PDA\mathcal{S}_{ij}, \quad \mathcal{S}N_i = \sum\nolimits_{j=1}^{n} \zeta_j NDA\mathcal{S}_{ij}$$

Step 7: Normalized the $\mathcal{S}P_i$ and $\mathcal{S}N_i$ by using the following formula:

$$N\mathcal{S}P_i = \frac{\mathcal{S}P_i}{\max_i(\mathcal{S}P_i)}, \quad N\mathcal{S}N_i = 1 - \frac{\mathcal{S}N_i}{\max_i(\mathcal{S}N_i)}$$

Step 8: Based on $N\mathcal{S}P_i$ and $N\mathcal{S}N_i$, calculate the appraisal score ($A\mathcal{S}$) value by using the following formula:

$$A\mathcal{S}_i = \frac{1}{2}(N\mathcal{S}P_i + N\mathcal{S}N_i)$$

Step 9: Depend on the value of $A\mathcal{S}_i$, rank all the values in specific order. Larger the value of $A\mathcal{S}_i$ superior that value is.

VII. ILLUSTRATIVE EXAMPLE BASED ON EDA$\mathcal{S}$ METHOD

To show the efficiency and superiority of investigated approach we will present a practical MCGDM example of small hydropower plant (SHPP) which is cited from [71].

Consider a construction company launched a project of four SHPP $\{\wp_1, \wp_2, \wp_3, \wp_4\}$ in different geographical sites of Pakistan that are consider for further evaluation to choose the best optimal power plant for construction activities.

TABLE 5. IF rough aggregated decision matrix by using IFRWA operators.

	c_1	c_2	c_3
$\wp_1$	((0.6414,0.1375),(0.4879,0.1537))	((0.8000,0.1636),(0.6320,0.2667))	((0.7601,0.2182),(0.5908,0.1790))
$\wp_2$	((0.5627,0.1591),(0.6437,0.2434))	((0.7245,0.1856),(0.4000,0.2624))	((0.4402,0.1701),(0.5374,0.1976))
$\wp_3$	((0.5756,0.2445),(0.4161,0.2614))	((0.7300,0.1729),(0.6833,0.2209))	((0.4977,0.2182),(0.3698,0.3904))
$\wp_4$	((0.5689,0.1701),(0.5967,0.2333))	((0.5103,0.3144),(0.4309,0.4244))	((0.2724,0.1301),(0.3749,0.2933))

	c_4	c_5
$\wp_1$	((0.6865,0.1301),(0.6096,0.1757))	((0.7925,0.1000),(0.6792,0.1591))
$\wp_2$	((0.6229,0.2667),(0.4161,0.1906))	((0.6421,0.1908),(0.4666,0.2543))
$\wp_3$	((0.5654,0.3944),(0.4707,0.3579))	((0.5694,0.3638),(0.3306,0.1790))
$\wp_4$	((0.4721,0.1879),(0.3726,0.3530))	((0.5022,0.2572),(0.3844,0.2667))

TABLE 6. The value of average solution ($Av\mathcal{S}$).

c_1	((0.5884,0.1737),(0.5447,0.2185))
c_2	((0.7078,0.2016),(0.5534,0.2846))
c_3	((0.5293,0.1802),(0.4774,0.2522))
c_4	((0.5942,0.2252),(0.4755,0.2549))
c_5	((0.6448,0.2056),(0.4847,0.2096))

TABLE 7. The results of $PDA\mathcal{S}_{ij}$ matrix.

	c_1	c_2	c_3	c_4	c_5
$\wp_1$	0.0355	0.0817	0.1474	0. 1547	0.1836
$\wp_2$	0. 0230	0.0000	0. 0138	0. 0000	0.0000
$\wp_3$	0. 0000	0.0881	0.0000	0.0000	0.0000
$\wp_4$	0.0077	0.000	0.0000	0.000	0.0000

To assess the SHPP a construction company invited three professional experts $\mathcal{E}_i$ $(i = 1, 2, 3)$ with weight vector $w = (0.29, 0.33, 0.38)^T$. The experts assessed these four SHPP concerning the five criteria, which are c_1 = approachability, c_2 = socioeconomic climate, c_3 = constructability, c_4 = technical feasibility and c_5 = purchasing and feed-in tariffs with weight vector $\zeta = (0.21, 0.24, 0.22, 0.15, 0.18)^T$.

TABLE 8. The results of $NDA\mathcal{S}_{ij}$ matrix.

	c_1	c_2	c_3	c_4	c_5
$\wp_1$	0.0000	0.0000	0.0000	0.0000	0.0000
$\wp_2$	0.0000	0.0355	0.0000	0.0028	0.0187
$\wp_3$	0.0931	0.0000	0.1225	0. 1181	0. 31105
$\wp_4$	0.0000	0.2064	0.1362	0.1103	0.12905

TABLE 9. The results of $\mathcal{S}P_i$ and $\mathcal{S}N_i$ $(i = 1, 2, 3, 4)$.

$\mathcal{S}P_1 = 0.11057$	$\mathcal{S}N_1 = 0.0000$
$\mathcal{S}P_2 = 0.0079$	$\mathcal{S}N_2 = 0.0123$
$\mathcal{S}P_3 = 0.0212$	$\mathcal{S}N_3 = 0.0879$
$\mathcal{S}P_4 = 0.0016$	$\mathcal{S}N_4 = 0.1194$

The professional experts assessed their assessment report for each $\wp_i$ against their corresponding criteria in the form of IFRVs. Now using the developed approach of IFRWA operator to get the best SHPP system by utilizing the above step wise decision algorithm of EDA$\mathcal{S}$ method.

Step 1: Collect the evaluation information of professional decision makers for each alternative $\wp_i$ against their criteria c_j and construct a decision matrix $\mathbb{M} = \left[\mathcal{I}\left(\mathcal{K}_{ij}^{l}\right)\right]_{m\times n}$ which is given in Tables 2 – 4:

TABLE 10. Ranking ordered of the proposed models.

Proposed operators based on EDA$\mathcal{S}$ method	Appraisal score $v$alues of alternatives				Ranking
	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	
IFRWA	1.0000,	0.4824,	0.2232,	0.00702	$\wp_1 > \wp_2 > \wp_3 > \wp_4$
IFROWA	1.0000,	0.5351,	0.2507,	0.0000	$\wp_1 > \wp_2 > \wp_3 > \wp_4$
IFRHA	1.0000,	0.6181,	0.2559,	0.0000	$\wp_1 > \wp_2 > \wp_3 > \wp_4$
IFRWG	1.0000,	0.3864,	0.0382,	0.0207	$\wp_1 > \wp_2 > \wp_3 > \wp_4$
IFROWG	1.0000,	0.5330,	0.2684,	0.0000	$\wp_1 > \wp_2 > \wp_3 > \wp_4$
IFRHG	1.0000,	0.4321,	0.3285,	0.0000	$\wp_1 > \wp_2 > \wp_3 > \wp_4$

TABLE 11. Comparative study of proposed method with existing methods.

Methods	Appraisal score $v$alues of alternatives				Ranking
	$\wp_1$	$\wp_2$	$\wp_3$	$\wp_4$	
IFWA [3]	Inaccessible				×
IFWG [4]	Inaccessible				×
IFDWA [11]	Inaccessible				×
IFWDG [11]	Inaccessible				×
IFHWA [12]	Inaccessible				×
IFR[28,29,35]/IFRSS [34]	Inaccessible				×
IF- EDA$\mathcal{S}$ method [51]	Inaccessible				×
IF-TOPSIS method [35]	Inaccessible				×
IF-VIKOR method [44]	Inaccessible				×
IF-GRA method [45]	Inaccessible				×
IFRWA	1.0000,	0.4824,	0.2232,	0.00702	$\wp_1 > \wp_2 > \wp_3 > \wp_4$
IFROWA	1.0000,	0.5351,	0.2507,	0.0000	$\wp_1 > \wp_2 > \wp_3 > \wp_4$
IFRHA	1.0000,	0.6181,	0.2559,	0.0000	$\wp_1 > \wp_2 > \wp_3 > \wp_4$
IFRWG	1.0000,	0.3864,	0.0382,	0.0207	$\wp_1 > \wp_2 > \wp_3 > \wp_4$
IFROWG	1.0000,	0.5330,	0.2684,	0.0000	$\wp_1 > \wp_2 > \wp_3 > \wp_4$
IFRHG	1.0000,	0.4321,	0.3285,	0.0000	$\wp_1 > \wp_2 > \wp_3 > \wp_4$

Step 2: The collective information decision makers against their weight vector are aggregated by using the IFRWA operators to get the aggregated decision matrix $\mathbb{M} = \left[\mathfrak{I}\left(\mathcal{K}_{ij}\right)\right]_{(m\times n)}$ and the result is given in Table 5.

Step 3: All criteria are benefit types so need to normalize it.

Step 4: Determine the value of $Av\mathcal{S}$ by applying proposed approached for all alternative under each criteria is given in Table 6.

Step 5: Based on determined $Av\mathcal{S}$ as given in Table 6, we can find the score value of $Av\mathcal{S}_i$ $(i = 1, 2, \ldots 5)$ and then calculate $PDA\mathcal{S}$ and $NDA\mathcal{S}$ as given in Tables 7 – 8.

$$\mathcal{S}(Av\mathcal{S}_1) = 0.6852, \quad \mathcal{S}(Av\mathcal{S}_2) = 0.6938,$$
$$\mathcal{S}(Av\mathcal{S}_3) = 0.6436, \quad \mathcal{S}(Av\mathcal{S}_4) = 0.6474,$$
$$\mathcal{S}(Av\mathcal{S}_5) = 0.6786$$

Step 6: Next to calculate the $\mathcal{S}P_i$ and $\mathcal{S}N_i$ by using criteria weight vector $\zeta = (0.21, 0.24, 0.22, 0.15, 0.18)^T$, which is given in Table 9.

Step 7: Now to normalize the $\mathcal{S}P_i$ and $\mathcal{S}N_i$, as given below.

$$N\mathcal{S}P_1 = 1.0000, \quad N\mathcal{S}P_2 = 0.0680,$$
$$N\mathcal{S}P_3 = 0.1822, \quad N\mathcal{S}P_4 = 0.0140$$
$$N\mathcal{S}N_1 = 1.0000, \quad N\mathcal{S}N_2 = 0.8968,$$
$$N\mathcal{S}N_3 = 0.2636, \quad N\mathcal{S}N_4 = 0.0000$$

Step 8: Based on $N\mathcal{S}P_i$ and $N\mathcal{S}N_i$, now to calculate the appraisal score ($A\mathcal{S}$) value as:

$$A\mathcal{S}_1 = 1.0000, \quad A\mathcal{S}_2 = 0.4824,$$
$$A\mathcal{S}_3 = 0.2232, \quad A\mathcal{S}_4 = 0.00702$$

Step 9: Depend on the above calculation the ranking result of the proposed models based on EDA$\mathcal{S}$ method are given Table 10. From Table 10, it is clear that the ranking ordered is slightly different but the best optimal option for the proposed models remain same. Hence the company should select the best SHPP $\wp_1$.

A. COMPARATIVE STUDY

The EDA$\mathcal{S}$ method is based on $PDA\mathcal{S}$ and $NDA\mathcal{S}$ from $Av\mathcal{S}$. The superior value of $PDA\mathcal{S}$ and inferior value of $NDA\mathcal{S}$ is considered the optimal choice. Here, to show the superiority of our investigated IFR- EDA$\mathcal{S}$ method, a comparative analysis has been made with some existing methods in context (see [3, 4, 11, 12, 28, 29, 34, 35, 43, 44, 45, 51,]). Based on Table 5 with criteria weight vector$\zeta = (0.21, 0.24, 0.22, 0.15, 0.18)^T$, the aggregation results of comparative study of existing models with our method have been listed in Table 11. From Table 11, it is clear that the existing methods such as IF- EDA$\mathcal{S}$, IF-TOPSIS, IF-VIRKO, IF-GRA methods and some aggregation operators are inaccessible to solve the developed illustrated example of section 6 by using IF rough values. However, the methods presented in [28], [29], [34], [35] that have rough information but these methods are inaccessible to solve the proposed model. From the analysis of Table 11, we see that the existing methods have the deficiency of rough information and these approaches are not capable to solve and rank the developed example. Therefore, the developed approach is more capable and effective than the existing methods

B. CONCLUSION

The MCGDM has the high potential and discipline process to improve and evaluate multiple conflicting criteria in all areas of DM to get more satisfactory and feasible DM result. In DM problems, the factual information about some fact is usually unknown, and this uncertainty makes the decision process more challenging and complex. The primitive notions of rough sets and intuitionistic fuzzy set (IFS) are general mathematical tools having the ability to handle the uncertain and imprecise knowledge easily. EDA$\mathcal{S}$ method has a significant role in the decision making problems especially when more conflict criteria exist in MCGDM problems. This method is based on $PDA\mathcal{S}$ and $NDA\mathcal{S}$ from $Av\mathcal{S}$. Superior value of $PDA\mathcal{S}$ and inferior value of $NDA\mathcal{S}$ is considered the optimal choice. To study the hybrid structure of EDA$\mathcal{S}$ method with IFRVs, we get IFR-EDA$\mathcal{S}$ method. The aim of this manuscript is to present IFR- EDA$\mathcal{S}$ method based on IF rough averaging and geometric aggregation operators. In addition, we put forward the concept of IFRWA, IFROWA and IFRHA aggregation operators. Furthermore, the concept of IFRWG, IFROWG and IFRHG aggregation operators are investigated. The basic desirable characteristics of the developed operator are given in detail. A new score and accuracy functions are defined for the proposed operators. Next IFR-EDA$\mathcal{S}$ model for MCGDM and their stepwise algorithm are demonstrated by utilizing the proposed approach. Finally, a numerical example for the developed model is presented and a comparative study of the investigated models with some existing methods are expressed broadly which show that the investigated models are more effective and useful than the existing approaches.

In future work, we shall extend the proposed approach to different aggregation operators including Hamacher operations, Dombi operations, Choquet integral, Einstein operations, Maclaurin symmetric mean operators, interaction aggregation operators Bonferroni mean etc. with Intuitionistic and Pythagorean fuzzy information. We will also focus on the applications of the proposed method by using Intuitionistic and Pythagorean fuzzy information in different real-life problems. Moreover, we will extend the developed method to other generalization of fuzzy sets as well and apply it to other fields, such as medical diagnosis

DATA AVAILABILITY

The data used in this article are artificial and hypothetical, and anyone can use these data before prior permission by just citing this article.

CONFLICTS OF INTEREST

The authors declare that they have no conflicts of interest.

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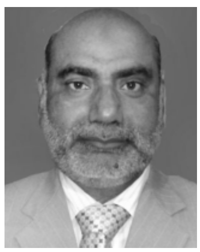

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