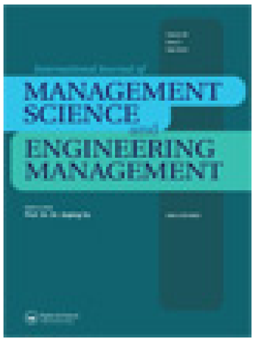

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Base-criterion on multi-criteria decision-making method and its applications

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ABSTRACT

In this paper, Base-criterion method (BCM) is proposed to solve multi-criteria decision-making problems. According to BCM, instead of executing a pairwise comparisons between all criteria or executing a pairwise comparisons between each of the best and worst criteria to other criteria, one of the criteria is chosen by the decision-maker as a base-criterion (e.g. preferential, selective) and then pairwise comparisons between base-criterion and other criteria are obtained. Then, a max-min problem is formulated and solved to determine the weight of the criteria. In this way, the pairwise comparisons to obtain weights of the criteria are fully consistent. In other ability of BCM, we find the lost pairwise comparisons by using the BCM framework so that the pairwise comparisons matrix remains consistent. To illustrate the reliability of the proposed method and evaluate its performance to determine the weights, we have used two numerical examples. The outcomes of numerical examples indicate that the BCM has high accuracy and better consistency ratio than the other multi-criteria decision-making(MCDM) methods. The cause of high consistency ratio in the BCM is the dependency of pairwise comparisons between criteria. Moreover, we have used two other numerical examples to illustrate the function of the BCM in order to find out the missing comparisons in incomplete pairwise comparison matrix. We have shown the BCM framework able to find lost comparisons in the worst terms of the incomplete pairwise comparison matrix. The outcomes of the proposed article show that BCM performance is significantly better than AHP and BWM methods with respect to the consistency ratio, and it requires fewer comparison data and has the ability to calculate missing pairwise comparisons.

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Multi-criteria decision making; multi-attribute decision making; pairwise comparison; incomplete data; consistency ratio

1. Introduction

Decision-maker can choice one alternative from a set of alternatives (Bhushan & Rai, 2004) based on some criteria. The decision makers face a situation in which they must choose from multiple alternatives (Yoon, 1987). In fact, decision-making problems are only made when there are multiple criteria and trade-offs (Zeleny, 2011). When multi-criteria are considered, decision-making can be called multi-criteria decision-making (MCDM) (Guo & Zhao, 2017; Triantaphyllou, 2000; Zeleny, 2011). MCDM (Dey, Bairagi, Sarkar, & Sanyal, 2015; Gonçalves, Dias, & Machado, 2015; Gurumurthy & Kodali, 2008) is a very important branch of decision-making theory. MCDM problems are generally divided into two categories of continuous and discrete problems. Multi-objective decision-making (MODM) methods to handle continuous problems, and multi-attribute decision-making (MADM) methods are used to handle discrete problems (Bhushan & Rai, 2004; Rezaei, 2015). According to the type of information available to the decision makings, MADM can be divided into four groups (Brauers, Zavadskas, Turskis, & Vilutiene, 2008; Keršuliene, Zavadskas, & Turskis, 2010; Ustinovichius, Zavadkas, & Podvezko, 2007):

In the first group, the rank correlation methods are consisting of total priorities ranking. Rank correlation was first introduced by Spearman (1987) and then this was taken over by Kendall and Berdahl (1970). The second group considers quantitative methods based on reference point or target such as the reference point method used in TOPSIS[1] (Tang, Shi, & Dong, 2019; Yoon, 1987), VIKOR[2] (Kang & Park, 2014; Mardani, Zavadskas, Govindan, AmatSenin, & Jusoh, 2016).COPRAS[3] (Zavadskas, Kaklauskas, Turskis, & Tamošaitiene, 2008) and Goal Programming (Broz, Vanzetti, Corsano, & Montagna, 2019; Huang, Yu, Chu, & Peng, 2017). In the third group, methods are based on quantitative measures. This group includes preference comparisons methods such as ELECTRE[4] (Mousavi, Gitinavard, & Mousavi, 2017; Roy, 1991; Yu, Zhang, Liao, & Qi, 2018) and PROMETHEE[5] (Brans & Vincke, 1985; Lolli et al., 2019; Sarrazin, De Smet, & Rosenfeld, 2018). The fourth group based on initial qualitative assessment, the results take a quantitative form at the next stage. This group includes methods such as AHP[6] (Saaty, 1977, 1988, 1990), ANP[7] (Chou, 2018; Saaty, 1996, 2005), Fuzzy Sets (Wang, Wang, & Zhang, 2019; Zadeh, 1965),Z-number (Aboutorab, Saberi, Asadabadi, Hussain, & Chang, 2018; Peng & Wang, 2018; Peng, Wang, Wang, & Wang, 2019), SWARA[8] (Keršuliene et al., 2010), and BWM[9] (Hafezalkotob & Hafezalkotob, 2017; Rezaei, 2015, 2016; Rezaei, Kothadiya, Tavasszy, & Kroesen, 2018). For the study and comparison of more MADM methods, this group is referred to the references (Gitinavard, Mousavi, & Vahdani, 2014; Li, Wang, & Hu, 2018; Mousavi, Gitinavard, & Siadat, 2014; Mousavi, Gitinavard, & Vahdani, 2015; Nie, Tian, Wang, Zhang, & Wang, 2018; Tavakkoli-Moghaddam, Gitinavard, Mousavi, & Siadat, 2015; Vahdani, 2016).

The problems dealing with the practical problems of MADM consist of two parts: one is to obtain the decision

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information including weights of the criteria and criteria values; and the other is collecting of criteria information and ranking the alternatives (Guo & Zhao, 2017). The objective of this proposed article is to find out the way through which the weights of the criteria can be obtained. Over the last decade, several MCDM methods have been proposed to obtain the weights of the criteria where AHP is one of the most used techniques. This method is applicable when criteria are independent.

In the AHP, the factors after selecting are arranged in a hierarchical structure descending from overall goal to criteria (C_1, C_2, …, C_n), sub-criteria (C_{11}, C_{12}, …, C_{21}, C_{22}, …, C_{nm})and alternatives (A_1, A_2, …, A_n) (Saaty, 1990). Then, the relative importance of criteria and alternatives is measured by an expert or team of experts by using pairwise comparison introduced by Thurstone's (1927) values on a scale of 1–9. Pairwise comparisons are used as a powerful inference tool and knowledge acquisition technique in knowledge-based systems. The practical and theoretical virtues of the pairwise comparison have its simplicity (Herman & Koczkodaj, 1996). The weights of the criteria (w_1, w_2, …, w_n) with condition $\mathrm{w_j} \geq 0$ and $\sum \mathrm{w_j} = 1$. The pairwise comparison scores are used in the same function to ranking the alternatives. The very important problem of the pairwise comparison method which usually occurs in practice is the incompatibility of the pairwise comparison matrix (Herman & Koczkodaj, 1996). The pairwise comparison matrix $\mathrm{A} = (\mathrm{a_{ij}})_{\mathrm{n*n}}$ is perfectly complete time for each i and j where $\mathrm{a_{ik}*a_{kj} = a_{ij}}$. Incompatibility in decision-making may have several reasons like decision-maker's lack of concentration (Forman & Selly, 2001). For this reason, despite the popularity and simplicity of the AHP, it is often unable to adequately handle the uncertainty of decision-maker preferences (Ishizaka & Nguyen, 2013).

The main cause of inconsistency is the unstructured way comparisons which are executed by pairwise comparisons-based method (Rezaei, 2015). Rezaei (2015) introduced the BWM and improved the consistency of pairwise comparisons by reducing the number of pairwise comparisons. In the BWM, first the best (e.g. most desirable, most important) and worst (e.g. least desirable, least important) criteria are identified by decision makers and then the relative importance of other criteria is measured against these two criteria (best and worst). Rezaei (2015), by comparing the BWM result and AHP, showed that the BWM results are more accurate. However, there is an inconsistency in BWM that can affect decision-making problems. In addition, sometimes it is difficult to identify the best and the worst criteria in the first step by the experts who make the pairwise comparisons and may have existed several best or worst criteria with the same importance. Another problem in the BWM is the lack of comparisons information between all criteria.

In our opinion, decision-makers are involved with a large amount of information and executing some of the secondary pairwise comparisons by an expert or a team of experts that cause inconsistency in the decision results.

In this paper, we propose a new MCDM method that obtains weights of the criteria based on a pairwise comparison in a different way, and it also needs a less number of pairwise comparisons than the existing MCDM methods. The inconsistency ratio in the proposed method is minimized so that the final matrix will be fully consistent. The remainder of this paper is organized as follows: In Section 2, a new MCDM method is proposed. In Section 3, the BCM is applied to incomplete pairwise comparison matrix problem, and it calculates missing comparison values. In Section 4, the conclusions and suggestions for future research are noted.

2. BCM

2.1. Direction and strength in pairwise comparisons

A pairwise comparison is the basis of our proposed method. Therefore, we try to make a better understanding of the pairwise comparisons. Suppose, there are n criteria, and we want to determine the relative importance of each criterion to other criteria. The matrix of pairwise comparisons will be as follows:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{mn} \end{bmatrix} \tag{1}$$

a_{ij} shows the relative importance of criterion i to the criterion j, which can be shown by using a numerical scale of 1/9 to 9. Similarly, a_{ji} shows the relative importance of criterion j to the criterion i which is reversely written as a_{ij} ($a_{ij} = 1/a_{ji}$). If $a_{ij} > 0$, criterion i is importance over criterion j and $a_{\mathrm{ij}} = 9$ represents the extreme importance of criterion i than the criterion j and $a_{\mathrm{ij}} = 1$ represents the equal importance between the criteria i and j.

In the pairwise comparisons between criteria, the direction and strength of the comparisons is questionable. In the proposed approach for pairwise comparisons in AHP, it is recommended to make a pairwise comparisons those are independent among all criteria. For example, if the relative importance of C_1 is stronger than C_2 and C_2 is also stronger than C_3then here is not required that the relative importance of C_1 takes stronger than C_3 (Saaty, 1977). On the other hand, the AHP discusses the consistency ratio which is a contradiction with independent pairwise comparison. In fact, this way disrupts the pairwise comparisons direction and leads to inconsistency. Several methods have been proposed to improve AHP consistency, none of which able to solve the basic problem of inconsistency (Harker, 1987a; Weiss & Rao, 1987). With these conditions, the pairwise comparisons among the criteria in the AHP method is ambiguous, and there is not exact relationship between the pairwise comparisons to obtain weights in terms of direction and strength. The weights obtained by this approach are not accurate and will have errors.

Unlike the pairwise comparison in AHP, secondary comparisons of the BWM are completely dependent. For example, if the relative importance of C_1 to C_2 be 4 and the relative importance of C_2 is equal to C_3, then the relative importance of C_1 to C_3 will be 4. With these conditions, if the relative importance of C_1 to C_3 instead of 4 is 3 or 5, an inconsistency occurs in the strength that is also observed in the secondary comparisons of BWM method.

n our proposed method (BCM), we consider the pairwise comparisons as dependent. We also believe that by performing a one-step pairwise comparison between a base (e.g. preferential, selective) criterion and other criteria, we can determine the relative importance of each criterion in

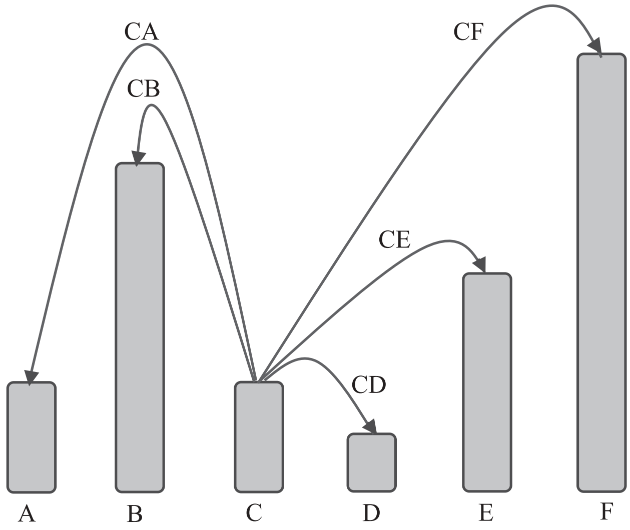

Figure 1. Base comparisons.

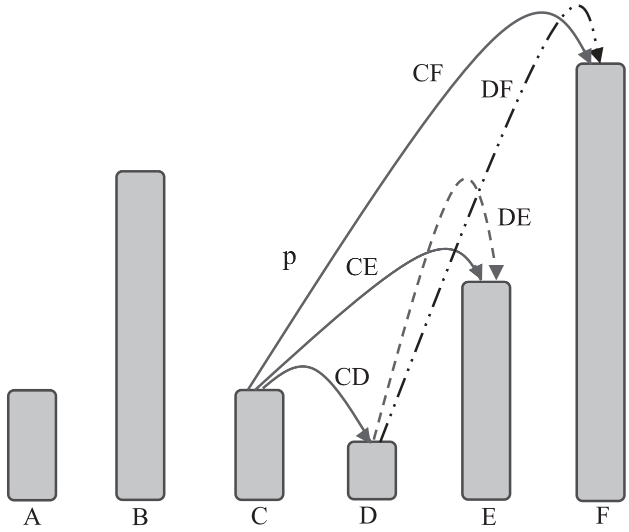

Figure 2. An example of the final comparisons.

relation to another criterion so that it is the perfect consistency in terms of direction and strength. For a better understanding, look at Figure 1. Schematic diagram of base comparisons is shown in Figure 2. Suppose there are six criteria and we want to get the weight of each criterion. To complete the pairwise comparison matrix, instead of executing the pairwise comparison among the criteria, we prefer to choose a criterion on the basis for comparisons (criterion *C* is chosen as the basis in Figure 1). Then, we determine the relative importance of the base-criterion against other criteria (lines CA, CB, CD, CE and CF in Figure 1) with a numerical scale of 1/9 to 9. Finally, in accordance with the base-comparisons, we use the following equation to determine the relative importance of each criterion to the other criteria.

$$a_{Base,i} \times a_{ij} = a_{Base,j} \tag{2}$$

In fact, pairwise comparisons are divided into two parts:

Definition 1. Comparison $a_{ij}$is defined as a base-comparison if i is the base-criterion.

Definition 2. The comparison a_{ij} is defined a final comparison if i and j are not base-criterion.

(1) Base Comparisons
(2) Final Comparisons

The final comparisons are derived from the base-comparisons performed by the decision-maker. In fact, the final comparisons are a subset of the base-comparisons. Firstly, decision-making is performed with base-comparisons. Without making the final comparisons, we can obtain the weight values of the criteria (See the next section for more details). Execution of final comparisons is also recommended to complete the matrix of pairwise comparisons as well as to determine the relative importance of a pairwise comparison among all criteria. See Figure 2 for a better understanding of the final comparisons. If we want to determine the relative importance of D to E (dashed line of DE in Figure 2), we can use two base-comparisons (lines CD and CE in Figure 2). For example, if the relative importance of CD is 2 and the relative importance of CE is 1/2, then the relative importance of D to E, which is the final comparison, is *1/4* $(a\text{Basec, C} * a\text{ij} = a\text{Base, D}; 2 * a\text{CD} = 1/2; a\text{CD} = 1/4)$. As a result, it is possible to obtain the value of each element having two elements of Equation (3).

An important point in the pairwise comparisons process is the allocation of logical numbers for comparisons. For example, if we want to assign a number for relative importance of C to D (line CD in Figure 1), the relative importance of C is obviously higher than D. Therefore, the decision-maker assigns a number greater than 1 to show relative importance of C to D. On the other hand, B, E and F are greater than C. So the decision-maker should assign a number for a CD that, in subsequent comparisons, does not violate the numerical scale of 1/9 to 9. The number 2 for the relative importance of CD is an appropriate number. If the decision-maker considers the numbers 3 or 4 for the CD, then it will contradict in the subsequent comparisons. If we assign 1/4 to the CF (in Figure 1), then the relative importance of D to F (dashed line DF in Figure 2) will be equal to 1/12 or 1/16. While numbers greater than 9 and less than 1/9 are not allowed for assignments to the comparisons.

Definition 3. The assignment of values for the strength of pair wise comparisons at this stage should be made by decision maker in some way that does not violate the final pair wise comparison values of the upper limit of the numerical scale 9 and the lower limit of 1/9.

According to Equation (2), the final-comparison values are obtained by dividing the values of the two base-comparisons. The values assigned to the base-comparisons are based on the numerical scale of 1/9 to 9. By dividing the base-comparisons values, it is possible that the result is out of the numerical scale of 1/9 to 9. In order to avoid this problem, the decision-maker should consider the model of Equation (3) during the assignment of values for the base-comparisons. The assignment of logical numbers for base-comparisons can be avoided by inconsistency in the strength. Here

$$a_{Base,i} \times a_{ij} = a_{Base,j}, a_{ij} = \frac{a_{Base,j}}{a_{Base,i}}$$

and (3)

$$\frac{1}{9} \le \frac{a_{Base,j}, j = 1, 2, \ldots, n}{a_{Base,i}, i = 1, 2, \ldots, n} \le 9 \ or \ \frac{1}{9} \le \text{aij} \le 9$$

In the next section, we shall show how the weight of the criteria can be obtained by base-comparisons.

2.2. Step of BCM

In this section, we describe the steps of BCM to obtain the weights of the criteria. Steps of BCM are as follows:

Step 1: Specify a set of decision criteria.

In this step, we consider the set of criteria (C_1, C_2, C_3, …, C_n) which are used for decision-making.

Step 2: Specify the base-criterion (e.g. preferential, selective).

Sometimes, identification the best or worst criteria in the first step is difficult and maybe a little difference or equal importance with several criteria as the best or worst of the criteria. That is why the decision-maker selects a preferential criterion as a base-criterion, but no comparison is made. For example, a decision-maker chooses the cost as a base-criterion among several criteria.

Step 3: Determine the relative importance of the base-criterion over the other criteria.

The relative importance of the pairwise comparisons will be shown at this step with a numerical scale of 1/9 to 9. The results of base-comparisons with other criteria are as follows:

$$a_{Base,j} \ (a_{B1}, a_{B2}, a_{B3}, \ldots, a_{Bn} \tag{4}$$

Where a_{Bj} indicates the base-criterion performance relative to j criterion. For example, this vector indicates the relative importance of the cost criterion over other criteria.

Step 4: Obtain the optimal weight of the criteria (w_1, w_2, …, w_n).

The optimal weights for w_B/w_j will be equal to a_{Bj}. For all j, we find a way to obtain the values of the maximum absolute difference $|w_B/w_j - a_{Bj}|$and take minimum of those values. Since the weight of the criteria is non-negative and aggregate, the problem can be expressed as follows:

$$\begin{aligned} &\text{Min max} \left| \frac{w_B}{w_j} - a_{Bj} \right| \\ &\text{Such that} \\ &\begin{cases} \sum_{j=1}^{n} R(w_j) = 1 \\ w_j \ge 0 \text{ for all } j \end{cases} \end{aligned} \tag{5}$$

Equation (5) can be rewritten as follows:

$$\begin{aligned} &\text{Min } \xi \\ &\text{Such that} \\ &\begin{cases} \left| \frac{w_B}{w_j} - a_{Bj} \right| \le \xi \\ \sum_{j=1}^{n} R(w_j) = 1 \\ w_j \ge 0 \text{ for all } j \end{cases} \end{aligned} \tag{6}$$

Note 1: The steps listed are a bit like the BWM with the difference that the achievement of the criteria weights is achieved by performing less steps and a simpler equation.

2.3. Consistency ratio

The consistency ratio is an important indicator to evaluate the degree of pairwise comparison matching. The comparisons are fully consistent if $a_{Base,i} \times a_{ij} = a_{Base,j}$, for all i and j. However, sometimes inconsistency in comparisons occurs for different reasons doing a lot of comparisons by decision-maker and lack of focus. In the BCM, the first step is executed as a base-comparison by decision-maker. Other pairwise comparisons (final-comparisons) are calculated by referring to the base-comparisons by Equation (2). For this reason, the value of ξ will be zero. According to Equation (7), all pairwise comparisons are fully consistent. Now

$$\text{Consistency Ratio} = \frac{\xi}{\text{Consistency Index}} \tag{7}$$

Pairwise comparisons in the BCM under two conditions are possible but not be fully consistent because of ignoring principle (3) by decision-maker in executing base-comparisons and errors in computing the final comparisons through Equation (2).

2.4. Numerical examples

In this section, two numerical examples are considered to justify BCM.

Example 1:A company needs to select an optimal transportation mode to deliver the products to a market. Rezaei (2015) used the BWM method to tackle this issue. For comparison of results, we adopt the transportation mode selection mentioned in (Rezaei, 2015) as the example 1 in this paper. The three criteria for this company are selected as Load Flexibility (C_1), accessibility (C_2) and Cost (C_3) (Step 1). The cost is chosen as the base-criterion (step 2). Then, pair wise comparisons are executed to determine the relative importance of the base-criterion to the other criteria (Step 3). Table 1 has shown the pair wise comparison of the base-criterion with the other criteria.

Based on the analysis of the information obtained from Table 1 and to find the weight of the criteria, the problem of limited nonlinear optimization can be built according to Equation (6) (Step 4).

Min ξ

Such that

$$\begin{cases} \left| \frac{w_3}{w_1} - 8 \right| \le \xi \\ \left| \frac{w_3}{w_2} - 2 \right| \le \xi \\ w_1 + w_2 + w_3 = 1 \\ w_1, w_2, w_3 \ge 0 \end{cases} \tag{8}$$

Table 1. Criteria.

Criteria	C_1	C_2	C3
Base-Criterion C_3	8	2	1

By solving Equation (8), the optimal weights of three criteria (load flexibility, accessibility and cost) can be calculated as follows:

$$\xi = 0, w_1 = 0.07692308,\ w_2 = 0.3076923, w_3 = 0.6153846$$

The weight of three criteria of load flexibility, accessibility and cost are equal to 0.0769, 0.3076 and 0.6153, respectively. Since in this relation $\xi = 0$, regardless of any value for the consistency index, the consistency ratio is the maximum because the consistency ratio is the closest number to zero.

$$w_1 \times 8 = w_3 \ \rightarrow\ 0.07692308 \times 8 = 0.6153846\ \xi = 0$$

$$w_2 \times 2 = w_3 \ \rightarrow\ 0.3076923 \times 2 = 0.6153846\ \xi = 0$$

The consistency ratio obtained above reflects the high accuracy of the BCM in determining the weights of the criteria. By comparing the weights obtained by BCM with BWM, we understand that the weights obtained with BWM are not accurate. For a better understanding of this issue, we show the weights obtained with BWM as follows. According to Rezaei (2015), the relative importance of w_3 to w_1 is 8. But, we compare the weights obtained by BWM, there is some difference between the weights which is inconsistent by the relative importance determined in the pairwise comparison. This difference between the weights is due to the error $\xi = 0.26$. For example, the weight value of the cost criterion (w_3) must be 8 times the weight of the load flexibility (w_1) which are shown as follows:

$$\mathrm{BWM}: A_{31} = 8 \ \rightarrow\ w_1 \times 8 = w_3 \ \rightarrow\ 0.0714 \times 8 = 0.5712 \neq 0.589$$

$$\mathrm{BCM}: A_{31} = 8 \ \rightarrow\ w_1 \times 8 = w_3 \ \rightarrow\ 0.7692 \times 8 = 0.6153$$

The difference in weight values obtained in the BWM indicates the low accuracy of this method. However, the difference in the weights may be acceptable. The above results indicate that the BCM method has high accuracy and offers a better consistency ratio than all existing methods. The reason for fully consistent in BCM is considering the dependence of pairwise comparison between the criteria in this method.

As described above, the final comparisons are a subset of the base-comparisons and can be obtained on the weight of the criteria without making the final comparisons. In this example, by using Equation (2) we compute the final comparisons values for completing the pairwise comparison matrix. Table 2 indicates a complete matrix consisting of base-comparisons and final comparisons.

$$a_{Base,i} \times a_{ij} = a_{Base,j} \ \rightarrow a_{32} \times a_{21} = a_{31} \ \rightarrow 2 \times a_{21} = 8 \rightarrow a_{21} = 4$$

In Table 2, the relative importance of accessibility to load flexibility is calculated as an example and shows with an arrow. The other elements of matrix are also calculated in the same way.

Example 2: The simple example is used to show the relative importance of the criteria set out in Figure 1. Car selection is an important issue for families. The buyers mostly evaluate the alternatives with consideration of the six criteria: convenience (A), fuel consumption (B), safety (C), style (D), acceleration (E) and consumer price (F) and then decides to select a car. The convenience (A), fuel consumption (B), safety (C), style (D), acceleration (E) and consumer price (F) are selected for optimal car selection (Step 1). In step 2, safety (C) is selected as the base-criterion. Table 3 shows the vector of pair wise comparisons of the base-criterion relative to other criteria (Step 3).

The results of model 5 for selecting the optimal car can be written as follows (Step 4).

Min ξ

Such that

$$\begin{cases} \left|\frac{w_C}{w_A} - 1\right| \leq \xi \\ \left|\frac{w_C}{w_B} - 1/3\right| \leq \xi \\ \left|\frac{w_C}{w_D} - 2\right| \leq \xi \\ \left|\frac{w_C}{w_E} - 1/2\right| \leq \xi \\ \left|\frac{w_C}{w_F} - 1/4\right| \leq \xi \\ w_A + w_B + w_C + w_D + w_E + w_F = 1 \\ w_A, w_B, w_C, w_D, w_E, w_F \geq 0 \end{cases} \tag{9}$$

By solving Equation (9), weights of the criteria will be obtained with ξ = 0 as follows:

$$w_A = 0.08695672,\ w_B = 0.2608702, w_C = 0.08695672,$$
$$w_D = 0.04347836,\ w_E = 0.1739134, w_F = 0.3478246,$$

Similarly as before of Example 1, $\xi = 0$ which indicates the maximum stability and consistency ratio of the comparisons. Table 4 indicates a complete matrix consisting of base and final comparisons.

2.5. Comparative analysis

To determine the weight of the n criterion, n2 requires a pairwise comparison between the criteria. The number n comparison is reduced from these relationships because of the equality of relative importance of each criterion. The half of the matrix is written in reverse, so the n(n-1)/2 pairwise comparisons are done by decision makers in

Table 3. Vector of pairwise comparisons.

Criteria	A	B	C	D	E	F
Base-criterion (C)	1	1/3	1	2	1/2	1/4

Table 2. Pairwise comparison matrix.

Criteria	Load Flexibility	accessibility	Cost
Load Flexibility	1	1/4	1/8
accessibility	4	1	1/2
Cost	8	2	1

$$a_{Base,i} \times a_{ij} = a_{Base,j} \rightarrow a_{32} \times a_{21} = a_{31} \rightarrow 2 \times a_{21} = 8 \rightarrow a_{21} = 4$$

Table 4. Complete matrix.

Criteria	Convenience (A)	Fuel consumption (B)	Safety (C)	Style (D)	Acceleration (E)	Consumer price (F)
Convenience (A)	1	1/3	1	2	1/2	1/4
Fuel consumption (B)	3	1	3	6	3/2	3/4
Safety (C)	1	1/3	1	2	1/2	1/4
Style (D)	1/2	1/6	1/2	1	1/4	1/8
Acceleration (E)	2	2/3	2	4	1	1/2
Consumer price (F)	4	4/3	4	8	2	1

AHP. In the BWM, the number of pairwise comparisons are reduced compared to the AHP. Consequently, 2n pairwise comparison is used to determine the relative importance of the best and worst criterion against other criteria. Also, the three pairwise comparisons are reduced because of the pairwise comparison of two criteria with itself and a pairwise comparison of the best to the worst criterion due to the repetition. As a result, one can obtain the weights of the criteria by executing (2n-3) pairwise comparisons. In the BCM, we shall execute n pairwise comparisons to determine the relative importance of the basic criterion over other criteria. One pairwise comparison is also reduced due to the equality of the relative importance of the basic criterion compared to it. In fact, the BCM method with the execution of $(n-1)$ pairwise comparisons are able to obtain the weights of criteria.

3. Incomplete pairwise comparison matrix

As mentioned above, pairwise comparisons may be inconsistency or incomplete because of the large number of pairwise comparisons, time pressure, lack of the expertise or incomplete information. As a result, the inconsistency of the pairwise comparisons and the incomplete pairwise comparisons have become one of the major issues in MCDM. For the missing comparison estimations, several methods and models have been proposed (Harker, 1987a, 1987b; Wedley, 1993; Zgurovsky, Totsenko, & Tsyganok, 2004). One of the developed methods to evaluate missing comparisons is the Incomplete Pairwise Comparison (IPC) algorithm suggested by Harker (1987a). Basis of Harker method, values of some pairwise comparisons are determined and temporary weights are calculated. Then, missing comparisons are estimated by w_i/w_j formulae. The basic problem in Harker method is an unusable in group decision-making, especially when used a questionnaire. In implementation of this method, it is necessary to follow the process of comparison by the experts and decision-maker simultaneously. In fact, in order to find incomplete pairwise comparisons, it is again necessary to consider the opinions of decision-maker to determine some missing pairwise comparisons. In this paper, we find missing pairwise comparisons by using the BCM framework so that the matrix of pairwise comparison does not come out of constancy.

To complete the incomplete pairwise comparison matrix, the following steps are followed:

$$A = \begin{bmatrix} 1 & a_{12} & \times & \cdots & a_{1n} \\ a_{21} & 1 & a_{23} & \cdots & \times \\ \times & a_{32} & 1 & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & \times & a_{n3} & \cdots & 1 \end{bmatrix} \tag{10}$$

Step 1: Fill in the missing comparisons with unknown variables $x_1$and $1/x_1$; $x_2$and $1/x_2$; etc.

$$A = \begin{bmatrix} 1 & a_{12} & x_1 & \cdots & a_{1n} \\ a_{21} & 1 & a_{23} & \cdots & x_p \\ 1/x_1 & a_{32} & 1 & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & 1/x_p & a_{n3} & \cdots & 1 \end{bmatrix} \tag{11}$$

Step 2: Check the pairwise comparisons according to Equations (2) and (3), and correct the pairwise comparison values if there is a difference.

Step 3: Specify the row which has the largest number of missing pairwise comparisons.

Step 4: Select the row as base row (Base-criterion) which has the least missing pairwise comparisons.

Step 5: Calculate unknown variables using Equation (2).

Step 6: Check the pairwise comparisons considering Equation (3).

For better understanding, the steps to complete the incomplete pairwise comparison matrix, look at the following examples.

Example 3: We solve the incomplete pair wise comparisons matrix of (Xu & Wang, 2013) in this paper. Suppose there are five criteria C_1, C_2, C_3, C_4 and C_5 where the decision maker fills only a_{12}, a_{13}, a_{15}, a_{23}, a_{24}, a_{34}, and a_{45} due to the lack of related information, special restrictions and time pressure, and unknown comparisons a_{14}, a_{25} and a_{35}. The following matrix represents the incomplete pair wise comparison between the five criteria.

$$A = \begin{bmatrix} 1 & 3 & 6 & \times & 4 \\ 1/3 & 1 & 2 & 1 & \times \\ 1/6 & 1/2 & 1 & 1/2 & \times \\ \times & 1 & 2 & 1 & 2 \\ 1/4 & \times & \times & 1/2 & 1 \end{bmatrix}$$

As mentioned above, we calculate the missing comparisons using the BCM framework by the following steps.

Step 1: Fill in the missing comparisons with unknown variables x_1 and $1/x_1$; x_2 and $1/x_2$; etc.

$$A = \begin{bmatrix} 1 & 3 & 6 & x_1 & 4 \\ 1/3 & 1 & 2 & 1 & x_2 \\ 1/6 & 1/2 & 1 & 1/2 & x_3 \\ 1/x_1 & 1 & 2 & 1 & 2 \\ 1/4 & 1/x_2 & 1/x_3 & 1/2 & 1 \end{bmatrix}$$

Step 2: Check all pairwise comparisons. Given that $a_{42} = 2$ and $a_{45} = 2$, the relative importance of C_3 will be equal to C_5. In the above matrix, the relative importance of a_{13} is set to 6 while the relative importance of a_{15} is *4* that causes inconsistency. For better understanding and inconsistency correction, look at the relationships between the pairwise comparisons in rows 1 and 4.

$$\begin{cases} a_{42} = 1 \ and \ a_{12} = 3 \\ a_{43} = 2 \ and \ a_{13} = 6 \end{cases} \rightarrow C_3 \text{ equal to } C_5 \rightarrow \text{if } a_{45} = 2 \text{ and } a_{15} = 6$$

Therefore, the value of a_{15} in A is changed to 6 as follows:

$$A = \begin{bmatrix} 1 & 3 & 6 & x1 & 6 \\ 1/3 & 1 & 2 & 1 & x2 \\ 1/6 & 1/2 & 1 & 1/2 & x3 \\ 1/x1 & 1 & 2 & 1 & 2 \\ 1/6 & 1/x2 & 1/x3 & 1/2 & 1 \end{bmatrix}$$

Step 3: Row C_5 has the most unknown variables (two unknown variables).

Step 4: According to Step (2), we select two rows (in other words, criterion) as the base-criterion for doing comparisons. C_1, C_2, C_3 and C_4 can be selected as a base-criterion. We select the C_1 and C_2 as two base-criteria.

Step 5: We calculate the unknown variables using Equation (2) as follows:

$$a_{Base,i} \times a_{ij} = a_{Base,j}$$

$$x_1 : a_{21} \times a_{14} = a_{24} \rightarrow 1/3 \times a_{14} = 1 \rightarrow a_{14} = 3 \rightarrow x_1 = 3$$

$$x_2 : a_{12} \times a_{25} = a_{15} \rightarrow 3 \times a_{25} = 6 \rightarrow a_{25} = 2 \rightarrow x_2 = 2$$

$$x_3 : a_{13} \times a_{35} = a_{15} \rightarrow 6 \times a_{35} = 6 \rightarrow a_{35} = 1 \rightarrow x_3 = 1$$

The complete matrix of pairwise comparison is as follows:

$$A = \begin{bmatrix} 1 & 3 & 6 & 3 & 6 \\ 1/3 & 1 & 2 & 1 & 2 \\ 1/6 & 1/2 & 1 & 1/2 & 1 \\ 1/3 & 1 & 2 & 1 & 2 \\ 1/4 & 1/2 & 1 & 1/2 & 1 \end{bmatrix}$$

Step 6: The calculated values for the missing comparisons in accordance with Equation (3) are as follows:

$$x_1 : 1/9 \le 3 \le 9;$$
$$x_2 : 1/9 \le 2 \le 9;$$
$$x_3 : 1/9 \le 1 \le 9;$$

Definition 4: After completing the pairwise comparisons matrix, it does not matter which criterion is considered as the base-criterion as the matrix is consistent.In this case, each of the criteria is considered as the base-criterion and the results are the same.

(12) Example 4: Suppose there are six criteria A_1, A_2, A_3, A_4, A_5 and A_6 which should be prioritized in certain circumstances with incomplete information. In this example, we face with the worst terms of the incomplete pairwise comparison matrix as the decision maker only fills the values a_{14}, a_{16}, a_{23}, a_{25}, a_{35} and a_{46}. In other words, more than half of the necessary comparison values has been lost. In this case, only the use of Equation (2) can solve the problem because of the large number of unknown variables. Therefore, we must use a linear programming considering Equation (3) to calculate the results from Equation (2) where

$$B = \begin{bmatrix} 1 & \times & \times & 4 & \times & 1/2 \\ \times & 1 & 1/2 & \times & 1 & \times \\ \times & 2 & 1 & \times & 2 & \times \\ 1/4 & \times & \times & 1 & \times & 1/8 \\ \times & 1 & 1/2 & \times & 1 & \times \\ 2 & \times & \times & 8 & \times & 1 \end{bmatrix}$$

Step 1: Fill in the missing comparisons with unknown variables x_1 and $1/x_1$; x_2 and $1/x_2$; etc.

$$B = \begin{bmatrix} 1 & x_1 & x_2 & 4 & x_3 & 1/2 \\ 1/x_1 & 1 & 1/2 & x_4 & 1 & x_5 \\ 1/x_2 & 2 & 1 & x_6 & 2 & x_7 \\ 1/4 & 1/x_4 & 1/x_6 & 1 & x_8 & 1/8 \\ 1/x_3 & 1 & 1/2 & 1/x_8 & 1 & x_9 \\ 2 & 1/x_5 & 1/x_7 & 8 & 1/x_9 & 1 \end{bmatrix} \quad (13)$$

Step 2: Check all pairwise comparisons. All pairwise comparisons are consistent here.

Step 3: Observing the matrix, we find that all rows have three unknown variables (unknown variable of rows are equal).

Step 4: Given that each row contains three unknown variables, we select three rows (base-criterion) for base-comparisons. Since unknown variable of all rows are equal, we select three criteria to our preference (rows A_1, A_2 and A_3).

Step 5: Using Equation (2), we calculate the unknown variables which are shown in Table 5 as follows:

Based on the above analysis and to obtain unknown variables, the following nonlinearly constrained optimization problem can be built according to Equation (2).

Min ξ

Such that

$$\frac{1}{9} \le \frac{\text{Upper value in row } i}{\text{Lower value in row } i} \le 9$$

For each unknown variable, aij are

$$\begin{cases} a_{Base,i} \times a_{ij} = a_{Base,j} + \xi; \\ \frac{1}{9} \le a_{ij} \le 9; \\ \text{Lower value in row } i \le a_{ij} \le \text{Upper value in row } i \\ \text{Lower value in column } j \le a_{ij} \le \text{Upper value in column } j \\ \xi \ge 0; \end{cases} \quad (14)$$

Then, we obtain the following nonlinearly constrained optimization problem represented by concrete numbers as follows:

Min ξ

Such that

Table 5. Calculation of the unknown variables.

Base-Criterion	Unknown Variables
a_1	x_4: $a_{12} \times a_{24} = a_{14} \rightarrow x_1 \times a_{24} = 4 \rightarrow a_{24} = 4/x_1$
	x_5: $a_{12} \times a_{26} = a_{16} \rightarrow x_1 \times a_{26} = 1/2 \rightarrow a_{26} = 1/(2x_1)$
	x_6: $a_{13} \times a_{34} = a_{14} \rightarrow x_2 \times a_{34} = 4 \rightarrow a_{34} = 4/x_2$
	x_7: $a_{13} \times a_{36} = a_{16} \rightarrow x_2 \times a_{36} = 1/2 \rightarrow a_{36} = 1/(2x_2)$
	x_8: $a_{14} \times a_{45} = a_{15} \rightarrow 4 \times a_{45} = x_3 \rightarrow a_{45} = x_3/4$
	x_9: $a_{15} \times a_{56} = a_{16} \rightarrow x_3 \times a_{56} = 1/2 \rightarrow a_{56} = 1/(2x_3)$
a_2	x_3: $a_{21} \times a_{15} = a_{25} \rightarrow 1/x_1 \times a_{15} = 1 \rightarrow a_{15} = x_1$
	x_2: $a_{21} \times a_{13} = a_{23} \rightarrow 1/x_1 \times a_{13} = 2 \rightarrow a_{13} = x_1/2$
a_3	x_1: $a_{31} \times a_{12} = a_{32} \rightarrow 1/x_2 \times a_{12} = 2 \rightarrow a_{12} = 2x_2$

$$
\begin{cases}
x_1 - 2 * x_2 = \xi; \\
x_2 - \frac{x_1}{2} = \xi; \\
x_3 - x_1 = \xi; \\
x_4 - \frac{4}{x_1} = \xi; \\
x_5 - \frac{1}{2*x_1} = \xi; \\
x_6 - \frac{4}{x_2} = \xi; \\
x_7 - \frac{1}{2*x_2} = \xi; \\
x_8 - \frac{x_3}{4} = \xi; \\
x_9 - \frac{1}{2*x_3} = \xi; \\
\xi \geq 0; \\
x_1 \geq \frac{1}{2}; x_1 \leq 4; \\
x_2 \geq \frac{1}{2}; x_2 \leq 4; \\
x_3 \geq \frac{1}{2}; x_3 \leq 4; \\
x_4 \geq \frac{1}{4}; x_4 \leq 4; \\
x_5 \geq \frac{1}{4}; x_5 \leq 4; \\
x_6 \geq \frac{1}{4}; x_6 \leq 4; \\
x_7 \geq \frac{1}{4}; x_7 \leq 4; \\
x_8 \geq \frac{1}{4}; x_8 \leq 2; \\
x_9 \geq \frac{1}{4}; x_9 \leq 4;
\end{cases}
$$

Solving Equation (14), the unknown variables are calculated. The complete matrix of pairwise comparison is as follows:

$$
B = \begin{bmatrix}
1 & 2 & 1 & 4 & 2 & 1/2 \\
1/2 & 1 & 1/2 & 2 & 1 & 1/4 \\
1 & 2 & 1 & 4 & 2 & 1/2 \\
1/4 & 1/2 & 1/4 & 1 & 1/2 & 1/8 \\
1/2 & 1 & 1/2 & 2 & 1 & 1/4 \\
2 & 4 & 2 & 8 & 4 & 1
\end{bmatrix}
$$

Step 6: The calculated values for the comparisons are in accordance with Equation (3) as follows:

$$x_1 : 1/9 \leq 2 \leq 9$$
$$x_2 : 1/9 \leq 1 \leq 9$$
$$x_3 : 1/9 \leq 2 \leq 9$$
$$x_4 : 1/9 \leq 2 \leq 9$$
$$x_5 : 1/9 \leq 1/4 \leq 9$$
$$x_6 : 1/9 \leq 4 \leq 9$$
$$x_7 : 1/9 \leq 1/2 \leq 9$$
$$x_8 : 1/9 \leq 1/2 \leq 9$$
$$x_9 : 1/9 \leq 1/4 \leq 9$$

The above results show that all values of unknown variables are consistent.

4. Conclusion

In this paper, we propose a new MCDM method called BCM. In this method, one of the criteria is chosen first by the decision-maker as the base-criterion (e.g. selective, preferential), and then the weight of the criteria is obtained on the basis of a pairwise comparison between the base-criterion and the other criteria. To illustrate the applicability of BCM, we pose numerical examples of decision-making problem. In the first numerical example, the problem posed by Rezaei has been investigated for choosing an optimal transport mode (Rezaei, 2015). The results show that final weights derived from BCM are highly reliable than BWM. In the second numerical example, we create a real-world decision-making problem for selecting a car. The consistency ratio obtained in numerical examples shows that BCM performs better than other MCDM methods. We also have considered two other numerical examples to illustrate BCM applicability to find missing comparisons data in the incomplete pairwise comparisons matrix. BCM has several important features that make it a robust method to solve MCDM problems as follows:

- The final weights derived from BCM are very reliable because the comparisons are fully consistent while other MCDM methods have a low inconsistency ratio.
- BCM such as BWM is a vector-based method. But compared to BWM and AHP, it requires less number of pairwise comparisons. In BCM, we only need to have $(n-1)$ comparisons while $(2n-3)$ comparisons and $n(n-1)/2$ comparisons are needed in BWM and AHP, respectively.
- The scale 1 to 9 to determine the relative importance of all pairwise comparisons has made decision-makers confronted with limitations to determine the relative importance of comparing different criteria. In BCM, decision-makers will be able to assign suitable numbers to determine the relative importance of the criteria (for example, the fractional numbers are 9/5, which are between 1/9 and 9).
- BCM can be used to derive weight independently, and it can also be combined with other MCDM methods.
- In the BWM, a decision is made by a decision-maker and, if we take advantage of the views of several decision makers, there is a possibility of conflict in identifying the best and worst criteria. In BCM, after completing the pairwise comparisons matrix, it does not matter which criterion is considered as the base-criterion as the comparisons is fully consistent. In these conditions, each of the criteria is considered as the base-criterion and the results are the same.

For future directions and development of the BCM, the authors suggest to apply the BCM in some real-world decision-making problems and to compare the results with other MCDM methods for verifying and improving validation. In the future, we need to study the cases involved in groups of decision-makers with incomplete pairwise comparisons data to solve the decision-making problems. Finally, evaluating the opinions of decision makers using linguistic terms to determine the relative importance of criteria can be considered as an interesting study.

Notes

1. Technique for Order of Preference by Similarity to Ideal Solution.
2. VlseKriterijumskaOptimizacija I KompromisnoResenje.
3. Complex Proportional Assessment.
4. Elimination Et ChoixTraduisantlaREaite.
5. Preference Ranking Organization METHod for Enrichment Evaluations.
6. Analytic Hierarchy Process.
7. Analytic network process.
8. Step-wise Weight Assessment Ratio Analysis.
9. Best-Worst Method.

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