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DETERMINING OBJECTIVE WEIGHTS IN MULTIPLE CRITERIA PROBLEMS: THE CRITIC METHOD

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Scope and Purpose—Financial analysis provides a great variety of ratios for the evaluation of the firm's performance. Interfirm comparisons based on the consideration of multiple financial ratios can be treated with a multiple criteria decision method (MCDM). The relative importance of each ratio depends upon the scope of analysis performed and the analyst's subjective judgement. If an unbiased ranking of firms is wanted objective weights of importance are very useful. Objective weights are derived by quantifying the intrinsic information of each evaluation criterion. The purpose of this paper is to identify the sources of information carried by different criteria in MCDM problems and to develop a method for determining objective weights of importance. The advantages of using the proposed method for interfirm comparisons are highlighted by a case study of Greek pharmaceutical firms.

Abstract—The association of weights in multiple criteria problems is a critical stage of the whole decision making process. In some decision situations the extraction of subjective preferences is either difficult or undesirable. This paper proposes a method for the determination of objective weights which is based on the quantification of two fundamental notions of MCDM: the contrast intensity and the conflicting character of the evaluation criteria. The latter notion is of great importance in interfirm comparisons because the financial indices used are often highly correlated. The method developed is applied to a sample of industrial firms. The results are compared to those obtained by other sets of objective weights and show this method ensures a better compromise of the criteria examined.

1. INTRODUCTION

In today's competitive economic environment the ranking of firms creates much interest which is manifested in the publication of league tables constructed on the basis of financial ratios. Each financial ratio provides different information and does not permit an overall evaluation of the firms' performances. For this reason multicriteria methods of evaluation are often used in interfirm comparisons, since they can reflect the multidimensional character of modern enterprises [1–5].

The result expected from a multicriteria interfirm comparison is a cardinal or ordinal ranking of the firms, based on their performances with respect to the ratios included in the analysis. It is clear that the ranking obtained depends upon the weight of importance assigned to each financial ratio. The problem is that sometimes managers and financial analysts can not easily decide on the relative importance of ratios. Several methods developed for extracting the decision makers'

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judgement can help them in the assignment of weights [6–11]. However, each of these methods would elicit a different set of weights from the same decision maker. In addition, different decision makers interviewed with the same method address different weights, since they have a dissimilar perception of the relative significance of the criteria considered. This kind of discordance can hinder interfirm comparisons which often aim at the creation of a commonly accepted rankorder of firms.

Several methodological approaches have been proposed in order to handle similar decision situations. Firstly, there are methods requiring less information about the relative importance of the evaluation criteria [12]. Secondly, there are methods performing a sensitivity analysis of weights in order to demonstrate the stability of the results, thus assisting decision makers in expressing their own judgement [13–15]. These methods recognize the fact that each decision situation has some particular characteristics independent of the decision maker's way of thinking.

A more direct approach is based on determining objective weights without the intervention of any decision maker. "Attribute importance is as much a property of the attribute as it is of a decision maker" [16]. This statement indicates that attributes can be viewed as information sources and that weights of importance reflect the amount of information contained in each of them.

Zeleny relates this information concept to the contrast intensity characterizing each separate criterion. The standard deviation or an "entropy" measure of importance have been proposed for quantifying contrast intensity and thus deriving objective weights of criteria [16]. Both techniques assign weights that are higher, the more differentiated are the scores of the examined alternatives. The same reasoning seen from an opposite point of view, confirms that a criterion in which all alternatives have the same performance does not offer any additional information and it is useless to include it in the decision making process.

This paper adds a second dimension to the concept of information emitted by criteria in a multicriteria analysis. That of the conflict between different attributes. Conflict is a fundamental notion of Multiple Criteria Decision Making (MCDM) which constitutes the core of each decision situation. A multicriteria problem in which the performances of the alternatives in all evaluation criteria are in complete concordance, does not present any interest, as the choice is evident. In this case, the introduction of a new criterion providing a different ranking of the alternatives adds a significant amount of information and alters drastically the decision situation.

The notion of conflict is of primary importance in interfirm comparisons, since many financial ratios are often highly correlated. In particular, profitability ratios give rise to a pyramid of subsidiary ratios, each clarifying a different aspect of managerial effectiveness [17, 18]. The incorporation of several interdependent criteria could yield misleading results, while the arbitrary omission of some criteria entails the removal of more or less useful information sources.

A method for handling a similar decision situation is to aggregate highly correlated criteria into a single attribute which encompasses a multilateral information. Prinicpal Component Analysis (PCA) is a suitable technique to achieve such a unification, by using the first principal component of the highly correlated criteria [19]. However, PCA is not commonly used to rank objects, but rather to reduce the dimensionality in multivariate analysis.

The present paper presents the method CRITIC (CRiteria Importance Through Intercriteria Correlation), which aims at the determination of objective weights of relative importance in MCDM problems. The weights derived incorporate both contrast intensity and conflict which are contained in the structure of the decision problem. The method developed is based on the analytical investigation of the evaluation matrix for extracting all information contained in the evaluation criteria.

The method CRITIC, described in the next section, is applied to a sample of Greek pharmaceutical industries evaluated with regard to three of the most prominent indices of a firm's performance. An analytical procedure that permits for a comparative evaluation of the method proposed has been followed and the computational results illustrate the advantages of CRITIC, over other techniques aiming at an objective resolution of multicriteria decision problems. The concluding remarks included in the last section summarize the main findings of the study.

2. DESCRIPTION OF THE METHOD

For a finite set A of n alternatives and a given system of m evaluation criteria f_j, the multicriteria problem in its general form can be defined as follows:

$$\text{Max}\{f_1(a), f_2(a), \ldots, f_m(a)/a \in A\}. \tag{1}$$

For every criterion f_j of this multicriteria problem we define a membership function x_j mapping the values of f_j to the interval [0, 1]. This transformation is based on the concept of the ideal point. So, the value x_{aj} below, expresses the degree to which the alternative a is close to the ideal value f_j^*, which is the best performance in criterion j, and far from the anti-ideal value f_{j*}, which is the worst performance in criterion j. Both f_j^* and f_{j*}, are achieved by at least one of the alternatives under consideration.

$$x_{aj} = \frac{f_j(a) - f_{j*}}{f_j^* - f_{j*}}. \tag{2}$$

In this way the initial matrix of evaluations is converted into a matrix of relative scores with generic element x_{ij}. By examining the jth criterion in isolation we generate a vector x_j denoting the scores of all n alternatives considered.

$$x_j = (x_j(1), x_j(2), \ldots, x_j(n)). \tag{3}$$

Each vector x_j is characterized by the standard deviation, σ_j which quantifies the contrast intensity of the corresponding criterion. So, the standard deviation of x_j is a measure of the value of that criterion to the decision making process. It is clear that any other index of the divergence in scores (like entropy or variance) could be used instead of the standard deviation.

Next, a symmetric matrix is constructed, with dimension $m \times m$ and a generic element r_{jk}, which is the linear correlation coefficient between the vectors $\mathbf{x_j}$ and $\mathbf{x_k}$. It can be seen that the more discordant the scores of the alternatives in criteria j and k, the lower the value r_{jk}. In this sense, the sum shown in formula (4) represents a measure of the conflict created by criterion j with respect to the decision situation defined by the rest of criteria.

$$\sum_{k=1}^{m} (1 - r_{jk}). \tag{4}$$

It should be noticed that the Spearman rank correlation coefficient R_{jk}^s could be used instead of r_{jk} in order to provide a more general measure of the relationship connecting the rank orders of the elements included in the vectors $\mathbf{x_j}$ and $\mathbf{x_k}$.

As mentioned in Section 1, information contained in MCDM problems is related to both contrast intensity and conflict of the decision criteria. Hence, the amount of information C_j, emitted by the jth criterion can be determined by composing the measures which quantify the two notions through the following multiplicative aggregation formula:

$$C_j = \sigma_j \cdot \sum_{k=1}^{m} (1 - r_{jk}). \tag{5}$$

According to the previous analysis, the higher the value C_j, the larger the amount of information transmitted by the corresponding criterion and the higher its relative importance for the decision making process. Objective weights result by normalizing these values to unity according to the following equation:

$$w_j = \frac{C_j}{\sum_{k=1}^{m} C_k}. \tag{6}$$

3. CASE STUDY OF INTERFIRM COMPARISON

A sample of eight pharmaceutical firms, which cover 20% of the total demand in Greece has been selected for case study. These firms have been evaluated with respect to three ratios indicating the profitability, the market position and the labour productivity. It is widely recognized, both in theory and in practice, that these indices are placed among the most representative measures of

Table 1. Performances and ranking of firms

	Profitability		Market share		Productivity	
Firms	%	Ranking	%	Ranking	$million	Ranking
A	61.0	1	1.08	7	4.33	2
B	20.7	2	0.26	8	4.34	1
C	16.3	3	1.98	6	2.53	3
D	9.0	4	3.29	4	1.65	6
E	5.4	5	2.77	5	2.33	4
F	4.0	6	4.12	1	1.21	7
G	−6.1	7	3.52	2	2.10	5
H	−34.6	8	3.31	3	0.98	8

corporate success. The ratios considered are:

(1) Rate of return on capital employed: this is a typical profitability ratio indicating the creativity of the firm's total assets. It is considered that the primary objective of managerial efforts is that the firm continues to exist and to expand. To achieve this goal it must generate profits in order to cover the costs of staying in business and to supply the capital needed for innovation and expansion.
(2) Market share: the improvement of the firm's market position constitutes another major objective of industrial strategic planning. It is often used as the reference indicator in the construction of corporate league tables and it constitutes the most apparent measure of effectiveness for the comparative evaluation of firms producing similar products.
(3) Added value per employee: productivity ratios are intended to express how efficient is the conversion of inputs into outputs. Among them, labour productivity is considered to be a key-factor determining the firm's competitiveness.

Table 1 shows the performances of the firms examined on these three criteria as well as, the corresponding rank orders. It can be seen that the criteria of profitability and productivity present a relative concordance. In other words, firms with high values of labour productivity are in general more profitable than other, less productive firms. On the other hand, market share appears to be an independent criterion of corporate success.

4. APPLICATION AND METHOD EVALUATION

Three different approaches for determining objective weights are used to rank the firms under consideration:

(1) The method CRITIC. Objective weights are derived by using equations (5) and (6).
(2) A method based only on the contrast intensity of criteria, quantified by means of the standard deviation σ_j of the scores in criterion j (SD weights). Objective SD weights are derived by using equation (7).

$$w_j = \frac{\sigma_j}{\sum_{k=1}^{m} \sigma_k}. \tag{7}$$

(3) A calculation of mean weights (MW weights) based on the assumption that all criteria are of equal importance. Assignment of equal weights to the decision criteria reflects a completely neutral attitude of the decision maker and it is often considered that such an attitude guarantees the objectivity of the evaluation process. MW weights are computed by using the simple formula:

$$w_j = \frac{1}{m}. \tag{8}$$

These three different sets of weights are used to construct a multicriteria ranking of the firms

examined according to the following aggregation formula:

$$D_i = \sum_{j=1}^{m} w_j \cdot x_{ij} \tag{9}$$

where

D_i = the multicriteria score of firm i,
x_i = the score of firm i under criterion j,
w_j = the weight of criterion j, calculated according to the three different approaches previously listed.

For assessing the credibility of the method CRITIC, a comparative analysis of the three alternative rankings has been performed:

First, the three multicriteria rankings are compared, through the Spearman's rank correlation coefficient, to the unicriteria preorders defined in Table 1. This test allows for estimating in quantitative terms the relative contribution of each separate criterion to the multicriteria rankings.

Second, the three multicriteria rankings are compared to the ranking obtained if a PCA is applied to the highly correlated criteria. Namely, the indices of profitability and productivity which are highly correlated (see Section 5), are aggregated into one single criterion by means of their first principal component (PC_1) which is normalized to unity. Equal weights are then assigned to the composite criterion PC_1 and to the criterion of market share for obtaining a bicriteria ranking (called for abbreviation PCA) according to formula (9).

5. RESULTS AND DISCUSSION

By applying equation (2) we transform the performances shown in Table 1 into the score matrix $[x_{ij}]$ presented in Table 2. Elaboration of these data give:

(i) the standard deviation σ_j of the scores in each criterion,

$$\sigma_1 = 0.263 \qquad \sigma_2 = 0.321 \qquad \sigma_3 = 0.358$$

(ii) the correlation matrix $[r_{jk}]$ containing the values of linear correlation coefficients for each pair of criteria.

1	−0.656	0.811
−0.656	1	−0.934
0.811	−0.934	1

These results show that, in this specific case study, the criterion of productivity presents the highest contrast intensity, while the criterion of market share constitutes the major source of conflict, with profitability and productivity being highly correlated. Table 3 presents the objective weights produced for each of the three alternative approaches. It can be seen that the method CRITIC assigns a significantly higher weight to the criterion of market share, while SD weights do not differ significantly to each other, although productivity is assigned with a slightly higher value. The

Table 2. Source matrix

Firms	Profitiability	Market share	Productivity
A	1	0.212	0.996
B	0.578	0	1
C	0.524	0.446	0.460
D	0.455	0.785	0.200
E	0.418	0.650	0.401
F	0.404	1	0.069
G	0.298	0.845	0.335
H	0	0.790	0

Table 3. Objective weights of the evaluation criteria

Method	Profitability	Market share	Productivity
CRITIC	0.202	0.481	0.317
SD	0.279	0.341	0.380
MW	0.333	0.333	0.333

Table 4. Multicriteria rank orders

Firms	CRITIC	SD weights	MW weights
A	1	1	1
B	6	2	2
C	7	7	7
D	4	5	6
E	5	6	5
F	3	3	4
G	2	4	3
H	8	8	8

Table 5. Spearman's rank correlation coefficients between multicriteria and unicriteria rankings

Method	Profitability	Market share	Productivity
CRITIC	0.214	0.238	0.048
SD	0.571	−0.476	0.762
MW	0.525	−0.333	0.619

Table 6. PCA with respect to the criteria of profitability and productivity

	Eigenvalues	Eigenvectors		% Variance
PC1	0.2601	0.5655	0.8248	92
PC2	0.0191	0.8248	−0.5655	8

multicriteria rankings, shown in Table 4, are constructed for each set of weights according to the aggregation procedure defined.

To evaluate these three different rankings in terms of objectivity and ability to reflect the inherent information of the MCDM problem the simple tests previously mentioned are carried out.

Table 5 shows the values of the Spearman's rank correlation coefficients between multicriteria and unicriteria preorders. We observe that the ranking obtained by using the method CRITIC constitutes a more balanced solution which incorporates the amount of information provided by all evaluation criteria. On the contrary, in the case of SD and MW weights, although their values are more uniform, the result of the multicriteria analysis does not reflect information emitted by the criterion of market share. Since the underlying concept of multicriteria analysis is to find the best compromise between competing objectives, it can be deduced that CRITIC allows for a better resolution of the conflict characterizing a given decision situation.

Table 6 shows the results of PCA applied to the two criteria of profitability and productivity, which have been found to be highly correlated. It can be seen that the first principal component carries a high percentage of the total variance (92%). PC1 is calculated for all alternatives examined from the corresponding eigenvector and is normalized to unity as shown in Table 7. The resulting aggregated criterion is used in the bicriteria analysis together with the criterion of Market Share, both assigned with equal weights, in order to construct the firms ranking (last column of Table 7).

Table 8 presents the values of the Spearman's rank correlation coefficients between the three rankings under investigation and that obtained if a PCA is used as a first step of the multicriteria analysis. It can be seen that the correlation coefficient between CRITIC and PCA approximates

Table 7. PCA results

Firm	PC1	Normalized score	PCA ranking
A	1.387	1	1
B	1.1517	0.830	7
C	0.6757	0.487	6
D	0.4223	0.305	4
E	0.5671	0.409	5
F	0.2854	0.236	2
G	0.4448	0.316	3
H	0	0	8

Table 8. Spearman's rank correlation coefficients between milticriteria and PCA rankings

CRITIC	SD	MW
0.9524	0.6429	0.5952

unity which indicates that these approaches provide almost identical rankings. On the contrary, the two other sets of weights, do not exhibit a significant relationship with PCA.

6. CONCLUDING REMARKS

Objective weights of importance can be calculated by means of conventional statistical measures which characterize the matrix of evaluation. These measures represent in mathematical terms two fundamental concepts of MCDM: the contrast intensity of the alternatives' performances in each single criterion and the conflict of the evaluation criteria with each other. The extraction and exploitation of these two features which are stored as intrinsic information in the data defining the multicriteria problem, are beneficial to the decision making process.

Objective weights derived from the method CRITIC proposed in this paper are found to embody the information which is transmitted from all the criteria participating in the multicriteria problem. In addition objective weights offer an insight into the nature of the dilemmas created by the existence of conflicting criteria and enable the incorporation of interdependent criteria. The results of the method CRITIC are similar to those obtained when applying a PCA to aggregate highly correlated criteria. However, CRITIC is a more straightforward approach needing less computational effort. Besides, in the case of PCA crisp values of intercriteria correlation coefficients should be defined, in order to distinguish those criteria considered to be highly correlated. This subjective intervention is avoided by using the method CRITIC.

The application of the method CRITIC is recommended for interfirm comparisons carried out on the basis of multiple financial ratios. The strong relationships connecting many of these numerical indices of corporate performance can be identified without ignoring any additional information carried by each of them. The objective classification of the firms obtained by the method CRITIC can constitute a common point of reference which is necessary in many business activities (negotiations between different parties, agreement on future policies etc). Moreover, CRITIC can be applied in many other multicriteria problems in order to:

—define objective weights when a decision maker is non-existent,
—facilitate the decision maker in expressing his opinion on the relative importance of the criteria,
—reduce the subjective character of the decision making process, by composing subjective and objective weights in an index of overall importance.
—discard the non salient attributes, in a primary weighting of the evaluation criteria.

The method can be easily converted into an algorithmic form. Hence, it is appliable in a predecision stage, as a subroutine connected to any method of multicriteria ranking which requires the introduction of clear quantitative values reflecting the relative importance of the decision criteria.

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